GANESH INSTITUTE OF ENGINEERING AND TECHNOLOGY(GIET),
Jagannath Prasad, Andharua, Bhubaneswar

Linear Control System Engineering

(As per the 2025-26 syllabus of the SCTE&VT, Bhubaneswar, Odisha)

Fourth Semester

Electrical Engg.

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SUB:CONTROLSYSTEMENGINEERING(Th3)

LearningResources:

Sl.No	TitleoftheBook	Nameof Author	Publishers
1.	Control system	A.Anandakumar	PHI
2.	Control system	k. padmanavan	IK
3	Controlsystem Engineering	I.J.Nagarath,M.Gopal	WEN
4	Controlsystem engineering	ANatraj,RameshBabu	SCIENTIFIC

TOPICWISEDISTRIBUTIONPERIODS

Sl. No	Chapter	Topic	Periods as persyllabus	Periods Actually needed	Expected marks
01	01	Fundamentalofcontrolsystem	04	04	05
02	02	Mathematicalmodelofasystem	04	05	10
03	03	Controlsystemcomponents	04	04	05
04	04	Blockdiagramalgebra&Signalflow graph	08	09	15
05	05	TimeresponseAnalysis	10	11	15
06	06	AnalysisofstabilityByRoot Locus Technique	10	07	20
07	07	FrequencyresponseAnalysis	10	07	15
08	08	NyquistPlot	10	07	15
		TOTAL	**60**	**54**	**100**

CHAPTER-1

FUNDAMENTALOFCONTROLSYSTEM

INTRODUCTION

A control system is an arrangement of physical components connected or related in such a mannerastocommand,direct,orregulateitselforanothersystem,oristhatmeansbywhich any quantityof interestin a system is maintainedor alteredin accordancewith a desired manner.

Anycontrolsystemconsistsofthreeessentialcomponentsnamelyinput,systemandoutput. Theinputisthestimulusorexcitationappliedtoasystemfromanexternalenergysource.A system is the arrangement of physical components and output is the actual response obtained from the system.Thecontrolsystemmay be oneofthefollowingtype.

1) manmade
2) naturaland/orbiologicaland
3) hybridconsistingof manmadeandnaturalorbiological.

Examples:

1) Anelectricswitchismanmadecontrolsystem,controlling flow of electricity. input : flippingtheswitchon/off
   system : electricswitch
   output :flowornoflowofcurrent
2) Pointingafingeratanobjectisabiologicalcontrolsystem.
   input : direction of the object with respect to some direction system :consistsofeyes, arm,hand,fingerandbrainofamanoutput :actualpointeddirectionwith respect to same direction
3) Mandrivinganautomobileis a hybrid system. input :directionor lane
   system :drivershand,eyes, brain and vehicle output : headingof theautomobile.

1.1 ClassificationofControlSystems

Control systems are classified into two general categories based upon the controlactionwhichisresponsibletoactivatethesystemtoproducetheoutput viz.

1) Openloopcontrolsysteminwhichthecontrolactionisindependentoftheoutput.

2) Closedloopcontrolsysteminwhichthecontrolactionissomehow dependent uponthe outputandaregenerallycalledasfeedback controlsystems.

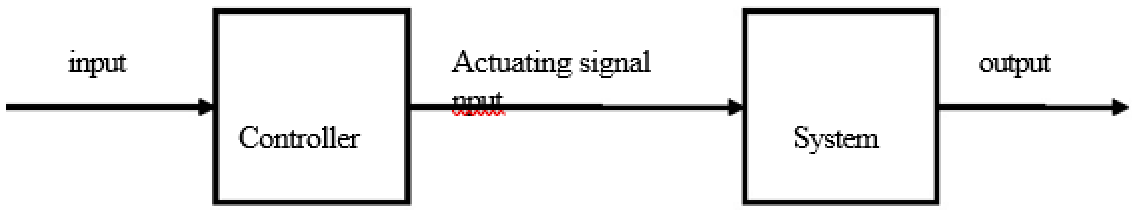
3)

4) **1.2 :Open Loop System: It**is a system in which control action is independentofoutput.Toeachreferenceinputthereisacorresponding outputwhichdependsuponthesystemanditsoperatingconditions.The accuracyofthesystemdependsonthecalibration ofthesystem.Inthe presence of noise or disturbances open loop control will not perform satisfactorily.

5)

6)

7)



8) EXAMPLE-1 Rotational Generator

9)

10) Theinputtorotationalgeneratoristhespeedoftheprimemover(e.gsteamturbine)in r.p.m.Assumingthegeneratorisonnoloadtheoutputmaybeinducedvoltageattheoutput terminals

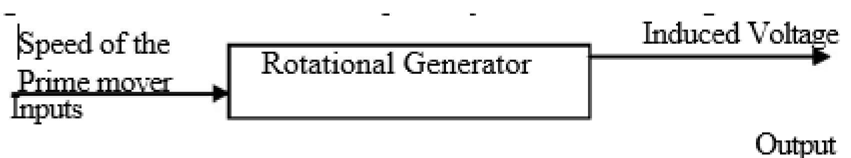


Fig 1-2 Rotational Generator

11) EXAMPLE-2Washingmachine

12) Most (but not all) washing machines are operated in the following manner. After the clothes to bewashed have been put into the machine, the soap or detergent, bleach and water areentered in proper amountsas specifiedbythemanufacturer.Thewashingtimeisthensetonatimerand thewasheris energized. Whenthecycleiscompleted,themachineshuts itselfoff.Inthisexamplewashingtimeformsinputandcleanlinessofthe clothes is identified as output.

13)

14)

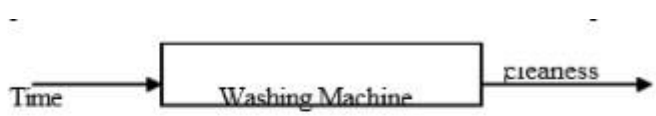
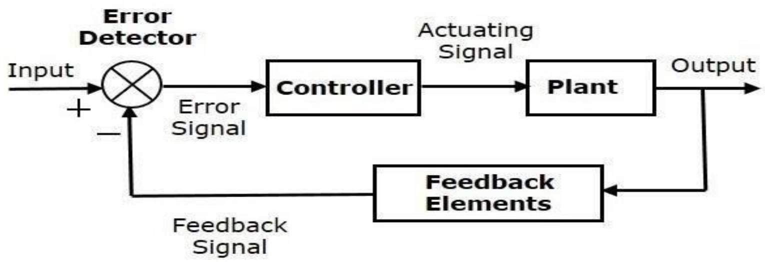


Fig 1-3 Washing Machine

15) CloseLoop

16) In **closed loop control systems**, output is fed back to the input. So,the control action is dependent on the desired output.

Thefollowingfigureshowstheblockdiagramofnegativefeedbackclosedloopcontrolsystem.



The errordetectorproduces an error signal, which isthe differencebetweenthe inputand the feedback signal. This feedback signal is obtained from the block (feedback elements) by consideringtheoutputoftheoverallsystemasaninputtothisblock.Insteadofthedirectinput, the error signal is applied as an input to a controller.

So,thecontrollerproducesanactuatingsignalwhichcontrolstheplant.Inthiscombination,the output of the control system isadjusted automatically till we get the desired response. Hence, the closed loop control systems are also called the automatic control systems. Traffic lights control system having sensor at the input isan example of a closed loop control system.

Thedifferencesbetweentheopenloopandtheclosedloopcontrolsystemsarementionedin the following tab

OpenLoopControlSystems	ClosedLoopControlSystems
Controlactionisindependentofthe desired output.	Controlactionisdependentof the desired output.
Feedbackpathisnot present.	Feedbackpath ispresent.
Thesearealsocalledasnon-feedback control systems.	These are also called as feedbackcontrolsystems.
Easy to design.	Difficulttodesign.
Theseareeconomical.	Thesearecostlier.
Inaccurate.	Accurate.

ControlSystems-Feedback

If eithertheoutputor some partof the output isreturnedtotheinput side andutilized aspart ofthesysteminput,then itisknownas **feedback**.Feedbackplaysanimportantroleinorderto improve the performance of the control systems. In this chapter, let us discuss the types of feedback & effects of feedback.

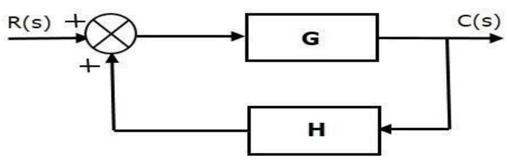
Typesof Feedback

Therearetwotypesoffeedback−

- Positivefeedback
- Negativefeedback

PositiveFeedback

The positivefeedbackadds the reference input, *R*(*s*) and feedbackoutput.The following figure shows the block diagram of **positive feedback control system**.



Theconceptoftransferfunctionwillbediscussedinlaterchapters.Forthetimebeing,consider the transfer function of positive feedback control system is,

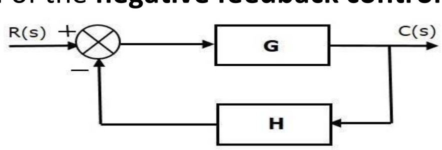
$$T = \frac{G}{1-GH} \qquad \text{(Equation 1)}$$

Where,

- **T**isthetransferfunctionoroverallgainofnegativefeedbackcontrolsystem.
- **G**istheopenloopgain,whichisfunctionoffrequency.
- **H**isthegainoffeedbackpath,whichisfunctionoffrequency.

Negative Feedback

Negativefeedbackreducestheerrorbetweenthereferenceinput, *R*(*s*)andsystemoutput.The following figure shows the block diagram of the **negative feedback control system**.



Transferfunctionofnegativefeedbackcontrolsystemis,

$$T = \frac{G}{1+GH} \qquad \text{(Equation 2)}$$

Where,

- **T**isthetransferfunctionoroverallgainofnegativefeedbackcontrolsystem.
- **G**istheopenloopgain,whichisfunctionoffrequency.
- **H**isthegainoffeedbackpath,whichisfunctionoffrequency.

1.3 :EffectsofFeedback

Letusnowunderstandtheeffectsoffeedback. Effect

of Feedback on Overall Gain

- FromEquation2,wecansaythattheoverallgainofnegativefeedbackclosedloop control system is the ratio of'G'and (1+GH). So, the overall gain may increase or decrease depending on the value of (1+GH).
- Ifthevalueof(1+GH)islessthan1,thentheoverallgainincreases. Inthiscase,'GH' value is negative because the gain of the feedback path is negative.
- Ifthevalueof(1+GH)isgreaterthan1,thentheoverallgaindecreases. Inthiscase,'GH' value is positive because the gain of the feedback path is positive.

Ingeneral,'G'and'H'arefunctionsoffrequency.So,thefeedbackwillincreasetheoverallgain of the system in one frequency range and decrease in the other frequency range.

1.4 :StandardTestSignals

Thestandardtestsignalsareimpulse,step,rampandparabolic.Thesesignalsareusedtoknowthe performance of the control systems using time response of the output.

Stepfunction

Aunitstepsignal,u(t)isdefinedas

$$u(t)=1; t \geq 0$$

$$u(t)=0; t<0$$

Following figure shows unit step signal.

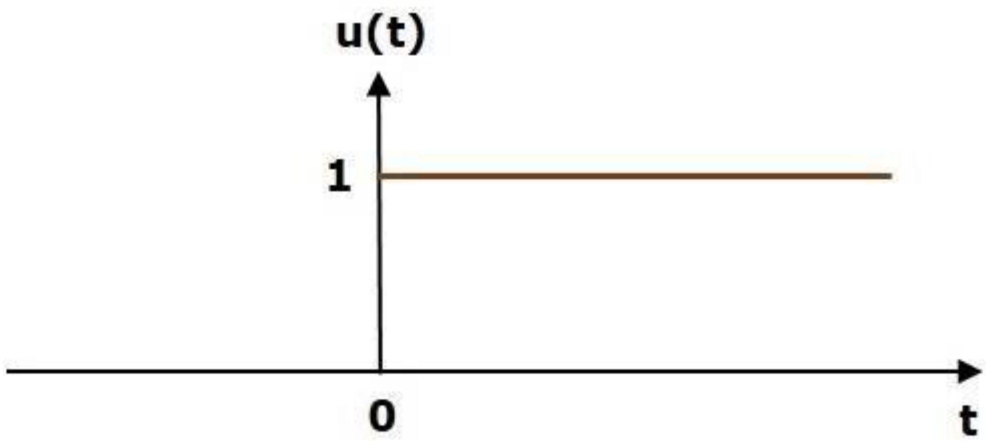
Stepfunction

Aunitstepsignal,u(t)isdefinedas

u(t)=1;t≥0

u(t)=0;t<0

Following figure shows unit step signal.



So,theunitstepfunctionexistsforallpositivevaluesof‘t’includingzero.Anditsvalueisoneduring this interval.Thevalueof theunitstepsignaliszeroforallnegativevaluesof‘t’.

UnitRamp function

Aunitrampsignal,r(t)isdefinedas

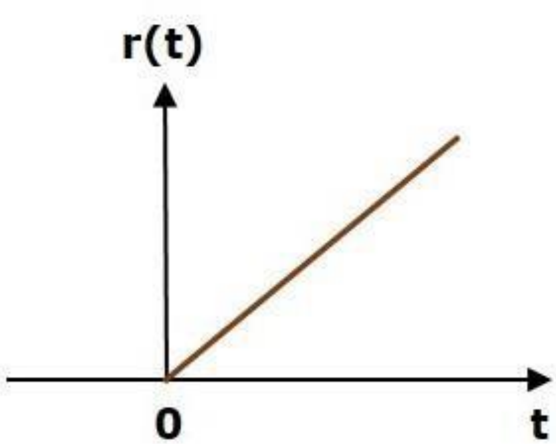
r(t)=t;t≥0

r(t)=0;t<0

Wecanwriteunitrampfunction,r(t)intermsofunitstep signal,u(t)as

r(t)=tu(t)

Followingfigureshowsunitrampsignal.



So, the unit ramp signal exists for all positive values of ‘t’ includingzero. And its value increases linearly with respect to ‘t’ during this interval. The value of unit ramp signal is zero for all negative values of ‘t’.

Parabolicfunction

Aunitparabolicsignal,p(t)isdefinedas,

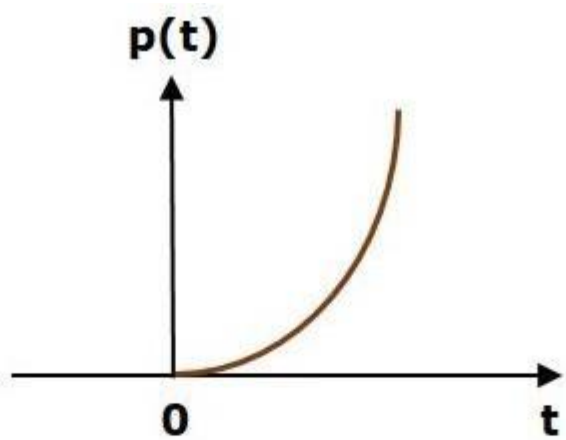
$$p(t)=t^2/2;t\geq 0$$

$$p(t)=0;t<0$$

Wecanwriteunitparabolicfunction, p(t)intermsoftheunitstep signal,u(t)as,

$$p(t)= p^2/2u(t)$$

Thefollowingfigureshowstheunitparabolic signal.



So, the unit parabolic signal exists for all the positive values of **‘t’**including zero. And its value increases non-linearlywithrespectto‘t’duringthisinterval.Thevalueoftheunitparabolicsignaliszeroforallthe negative values of ‘t’.

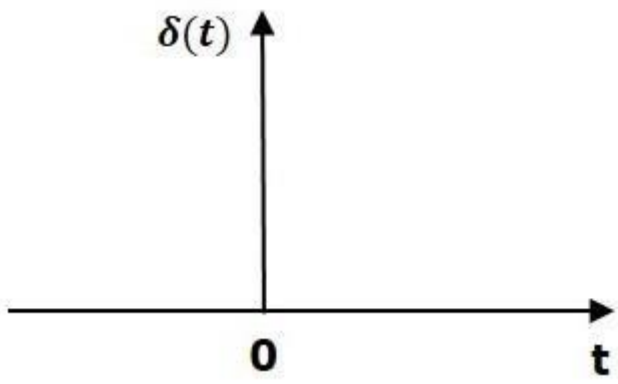
UnitImpulseSignal

Aunitimpulsesignal,δ(t)isdefinedas

$\delta(t)=0\delta(t)=0$ for $t\neq 0$

and $\int_{0^-}^{0^+}\delta(t)dt=1$

Thefollowingfigureshowsunitimpulse signal.



So,theunitimpulsesignalexistsonlyat‘t’isequaltozero.Theareaofthissignalundersmallintervalof timearound‘t’isequaltozeroisone.Thevalueofunitimpulsesignaliszeroforallothervaluesof‘t’.

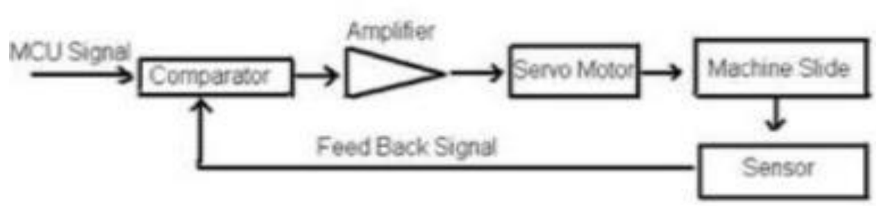
1.5 :Servomechanism

Servomechanism

Aservosystemprimarilyconsistsofthreebasiccomponents –acontrolleddevice,aoutput sensor,a feedback system.

This is an automatic closed loop control system. Here instead of controlling a device by applying the variableinputsignal,thedeviceiscontrolledbyafeedbacksignalgenerated bycomparingoutputsignal and reference input signal.
When referenceinput signal or command signal is applied tothe system, it is compared with output referencesignalofthesystemproducedbyoutputsensor,andathirdsignalproducedbyafeedback system.
This third signal acts as an input signal of controlled device.
Thisinputsignaltothedevicepresentsaslongasthereisalogicaldifferencebetweenreferenceinput signal and the output signal of the system.

After the device achieves its desired output, there will be no longer the logical difference between referenceinputsignalandreferenceoutputsignalofthesystem.Then,thethirdsignalproducedby comparing theses above said signals will not remain enough to operate the device further and to produce a further output of the system until the next reference input signal or command signal is applied to the system.



Shortquestions

1:Namethreeapplicationsofcontrolsystems.Ans:Guidedmissiles,Fighterplanestability,Satellitetracking antenna

2 WhatismeantbySystem?

Whenthenumberofelementsconnectedperformsaspecificfunctionthenthegroupofelementsissaidto constitute a system or interconnection of various components for a specific task is called system. Example: Automobile.

3 WhatismeantbyControlSystem?

Any set of mechanical or electronic devices that manages, regulates or commands the behavior of the system usingcontrolloopiscalledtheControlSystem.Itcanrangefromasmallcontrollingdevicetoalargeindustrial controlling device which is used for controlling processes or machines.

3 Whatisopenloopandcontrolloopsystems?

OpenloopcontrolSystem:Anopen-loopcontrolsystemisasysteminwhichthecontrolactionisindependentof the desired output signal. Examples: Automatic washing machine, Immersion rod.

ClosedloopcontrolSystem:Aclosed-loopcontrolsystem isasysteminwhichcontrolactionisdependentonthe desired output. Examples: Automatic electric iron, Servo voltage stabilizer, an air conditioner.

4 Whatarethenecessarycomponentsofthefeedbackcontrolsystem?

Theprocessingsystem(openloopsystem),feedbackpathelement,anerrordetector,andcontrollerarethe necessary components of the feedback control system.

5 Whatisthefeedbackinthecontrolsystem?

When the input is fed to the system and the output received is sampled, and the proportional signal is then fed backtotheinputforautomaticcorrectionoftheerrorforfurtherprocessingtogetthedesiredoutputiscalledas feedback in control system

Long Questions

1:What are the advantages and disadvantages of open loop control

System?2:Whataretheadvantagesanddisadvantagesofclosed-

loopcontrolSystem? 3:Name the three major design criteria for control

systems.

Chapter-2

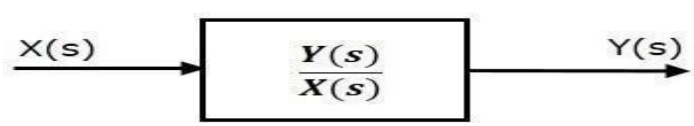
MathematicalmodelofaSystem

Thecontrolsystemscanberepresentedwithasetofmathematicalequationsknownas **mathematical model**. These models are useful for analysis and design of control systems. Analysisofcontrolsystemmeansfindingtheoutputwhenweknowtheinputandmathematical model. Design of control system means finding the mathematical model when we know the input and the output.

Thefollowingmathematicalmodelsaremostlyused.

- Differentialequationmodel
- Transferfunctionmodel

- # TransferFunctionModel

- Transfer function model is an s-domain mathematical model of control systems. The **Transfer function** of a Linear Time Invariant (LTI) system is defined as the ratio of Laplace transform of output and Laplace transform of input by assuming all the initial conditions are zero.
- If$x(t)$and $y(t)$aretheinputandoutputofanLTIsystem,thenthecorrespondingLaplace transforms are $X(s)$ and $Y(s)$.
- Therefore, the transfer function of LTI system is equal to the ratio of $Y(s)$ and $X(s)$. *i.e.*,$TransferFunction=Y(s)/X(s)$
- Thetransfer function modelofan LTIsystem is shownin the following figure.
- 

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- Here, we represented an LTI system with a block having transfer function inside it. And this block has an input $X(s)$ & output $Y(s)$.
-

The transfer function of a control system is defined as the ratio of the Laplace transformoftheoutputvariabletoLaplacetransformoftheinputvariableassuming all initial conditions to be zero.

Thusthecauseandeffectrelationshipbetweentheoutput and input is related to each other through a **transfer function**.

InaLaplaceTransform,iftheinputisrepresentedbyR(s)andtheoutputis represented by C(s), then the transfer function will be:

$$G(s) = \frac{C(s)}{R(s)} \Rightarrow R(s).G(s) = C(s)$$

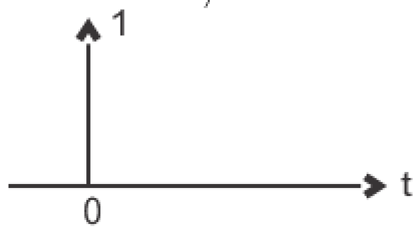
Thatis,thetransferfunctionofthesystemmultipliedbytheinputfunctiongivesthe output function of the system.

$$G(s) \;=\; \frac{C(s)}{R(s)}$$

TheEffectofImpulse Signal

Theunitimpulsesignalisdefined as

$$\delta(t) = 1 \; when \; t = 0$$
$$= 0 \; when \; t \neq 0$$



Laplacetransformofunitimpulsefunctionis1.

$$\pounds\delta(t) = 1$$

Now if inputsignalis unit impulse signal then,

$$\frac{C(s)}{R(s)} = G(s) \Rightarrow C(s) = R(s)G(s) \Rightarrow C(s) = G(s)$$

$$[\because \pounds\delta(t) = 1]$$

Theoutputfunctionissameasitstransferfunction.

PROPERTIESOFTRANSFERFUNCTION (TF)

Thepropertiesoftransferfunctionaregivenbelow:

- TheratioofLaplacetransformofoutputtoLaplacetransformofinputassumingall initial conditions to be zero.
- ThetransferfunctionofasystemistheLaplacetransformofitsimpulseresponse under assumption of zero initial conditions.

- Replacing‘s’variablewithlinearoperation $D \equiv \frac{d}{dt}$ intransferfunctionofasystem, the differential equation of the system can be obtained.
- Thetransferfunctionofasystemdoesnotdependontheinputstothesystem.
- Thesystempolesandzeroscanbedeterminedfromitstransferfunction.
- Stabilitycanbefoundfrom...characteristicsequations.

AdvantagesofTransferfunction

1. Iftransferfunctionofasystemisknown,theresponseofthesystemtoanyinputcanbe determined very easily.

2. Atransferfunctionisamathematicalmodelanditgivesthegainofthesystem.

3. SinceitinvolvestheLaplacetransform,thetermsaresimplealgebraicexpressionsand no differential terms are present.

4. Polesandzeroesofasystemcanbedeterminedfromtheknowledgeofthe transfer function of the system.

DisadvantagesofTransferfunction

1. Transferfunctiondoesnottakeintoaccounttheinitialconditions.

2. Thetransferfunctioncanbedefinedforlinearsystems only.

3. Noinferencescanbedrawnaboutthephysicalstructureofthesystem.

PolesandZerosofTransferFunction

Generally,a function can berepresented to its polynomialform.For example,

$$F(s) = f_0 s^n + f_1 s^{n-1} + f_2 s^{n-2} + f_3 s^{n-3} + \cdots\cdots + f_{n-1} s^1 + f_n$$

Nowsimilarlytransferfunctionofacontrolsystemcanalsoberepresentedas

$$G(s) = \frac{C(s)}{R(s)} = \frac{C_0 s^n + C_1 s^{n-1} + C_2 s^{n-2} + \cdots\cdots + C_{n-1} s + C_n}{R_0 s^m + R_1 s^{m-1} + R_2 s^{m-2} + \cdots + R_{m-1} s + R_m}$$

$$= K \frac{(s-z_1)(s-z_2)(s-z_3)\cdots(s-z_n)}{(s-p_1)(s-p_2)(s-p_3)\cdots(s-p_m)}$$

WhereKisknownasthegainfactorofthetransferfunction.

Now in the above function if s = z_1, or s = z_2, or s = z_3,….s = z_n, the value of transfer functionbecomeszero.Thesez_1,z_2,z_3,….z_n,arerootsofthenumeratorpolynomial.As for these roots the numerator polynomial, the transfer function becomes zero, these roots are called zeros of the transfer function.
Now,ifs= p_1,ors= p_2,or s= p_3,….s=p_m,thevalueoftransferfunctionbecomes infinite. Thus the roots of denominator are called the poles of the function.

Nowletusrewritethetransferfunctioninitspolynomialform.

$$G(s) = K\frac{(s-z_1)(s-z_2)(s-z_3)\cdots(s-z_n)}{(s-p_1)(s-p_2)(s-p_3)\cdots(s-p_m)}$$

Now,letusconsidersapproachestoinfinityastherootsareallfinitenumber,they can be ignored compared to the infinite s. Therefore

$$G(s) = K\frac{s^n}{s^m} = Ks^{n-m}$$

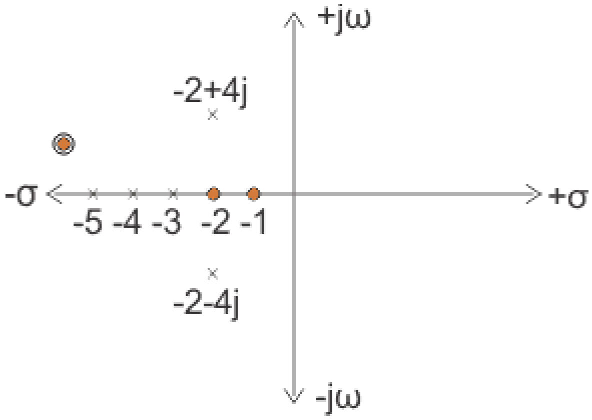
Hence, when s → ∞ and n > m, the function will have also value of infinity, that meansthetransferfunctionhaspolesatinfinites,andthemultiplicityororderof such pole is n – m.
Again, when s → ∞ and n < m, the transfer function will have value of zero that meansthetransferfunctionhaszerosatinfinites,andthemultiplicityororderof such zeros is m – n.

1) Letusexplaintheconceptofpolesandzerosoftransferfunctionthroughan example.

$$G(s) = \frac{(s+1)(s+2)}{(s+3)(s+4)(s+5)(s+2-4j)(s+2+4j)}$$

Solution
Thezerosofthefunctionare,-1,-2andthepolesofthefunctionsare-3,-4,-5,- 2 + 4j, -2 – 4j.

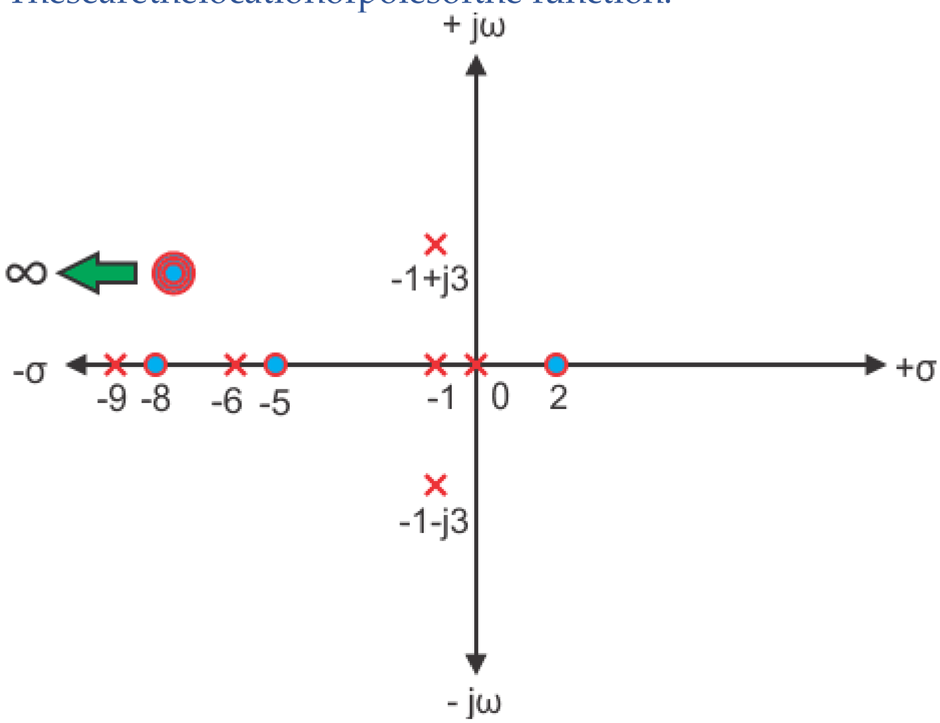
2) Heren=2 andm=5,asn< mandm– n= 3,thefunctionwillhave3zerosats →∞.Thepolesandzerosareplottedinthefigurebelow



2) Letustakeanotherexampleoftransferfunctionofcontrolsystem

$$G(s) = \frac{(s-2)(s+5)(s+8)}{s(s+1)(s+6)(s+9)(s+1-j3)(s+1+j3)}$$

Solution
Intheabovetransferfunction,ifthevalueofnumeratoriszero, then
$(s-2)(s+5)(s+8)=0$
$\Rightarrow s=2,\ -5,\ -8$
Thesearethelocationofzerosofthefunction.
Similarly,intheabovetransferfunction,ifthevalueofdenominatoriszero, then
$s(s+1)(s+6)(s+9)(s+1-j3)(s+1+j3)=0$
$\Rightarrow s=0,\ -1,\ -6,\ -9,\ -1+j3,\ -1-j3$
Thesearethelocationofpolesofthe function.



Asthenumberofzerosshouldbeequaltonumberofpoles,theremaining three zeros are located at s →∞.

MathematicalModellingofElectricalSystem

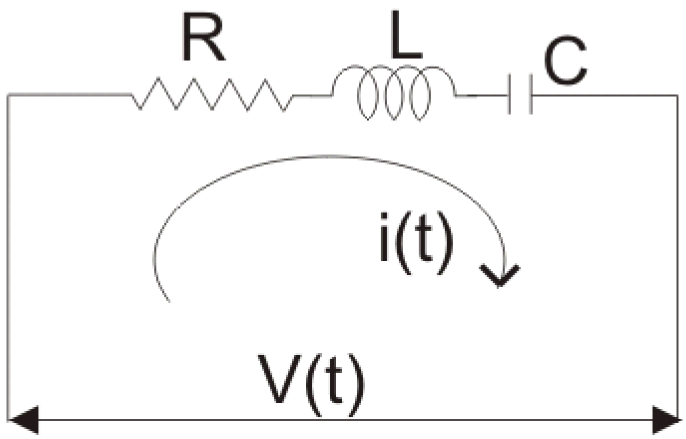
Inanelectricaltypeofsystemwehavethreevariables–

1. Voltagewhichisrepresentedby 'V'.
2. Currentwhichisrepresentedby 'I'.
3. Chargewhichisrepresentedby'Q'.

Andalsowehavethreeparameterswhichareactiveandpassivecomponents:

1. Resistancewhichisrepresentedby'R'.
2. Capacitancewhichisrepresentedby'C'.
3. Inductancewhichisrepresentedby 'L'.

Nowweareinconditiontoderiveanalogybetweenelectricalandmechanicaltypes of systems. There are two types of analogies and they are written below:
ForceVoltageAnalogy:Inordertounderstandthistypeofanalogy,letusconsidera circuit which consists of series combination of resistor, inductorand capacitor.



A voltageV is connected in series with these elements as shown in the circuit diagram.Nowfromthecircuitdiagramandwiththehelpof KVLequationwewrite the expression for voltage in terms of charge, resistance, capacitorand inductor as,

$$V = L\frac{dt^2}{d^2q} + R\frac{dt}{dq} + \frac{q}{C}$$

CHAPTER-3
CONTROLSYSTEM COMPONENTS

3.1 :componentofcontrolsystem

Transducerwhichisthefirstmajorcomponentinacontrolsystemisadevicethat sensestheoutputinoneformandconvertitintoanotherform,thesensingmaybe temperature,pressure,position,and conversion is generally into electrical

3.2 :Gyroscope

Itisaninstrumentusedinspaceshipsandaircrafts.Theinputistheangularvelocity andtheoutputistheangulardisplacement.Theactionofgyroscopeisisbasedon followingprinciples.

1. Ifnoexternaltorqueactsonitthespinningwheelmaintainsthe directionofitsspinaxisinspaceandthistypeofspinningisknownas freegyrotype
2. Iftorqueisappliedtoanaxisinclinedtothespinaxisofawheelthe wheelrotateaboutanaxisatanangle90^{0}toboththespinaxisaswell astheinputtorqueaxis.thistypeofrotationisknownasprecisiontype

Synchro

Definition: TheSynchroisa **typeof transducer** which **transforms**the**angularposition of theshaft** into an **electricsignal**.Itisused asan **errordetector**andasa**rotarypositionsensor.**Theerroroccursinthe system because of the misalignment of the shaft. The transmitter and the control transformer are the two main parts of the synchr

SynchrosSystemTypes

Thesynchrosystemisoftwotypes.Theyare

1. ControlTypeSynchro.
2. TorqueTransmissionTypeSynchro.

TorqueTransmissionTypeSynchros

Thistypeofsynchroshassmalloutputtorque,andhencetheyareusedforrunningtheverylightload like a pointer. The control type Synchro is used for driving the large loads.

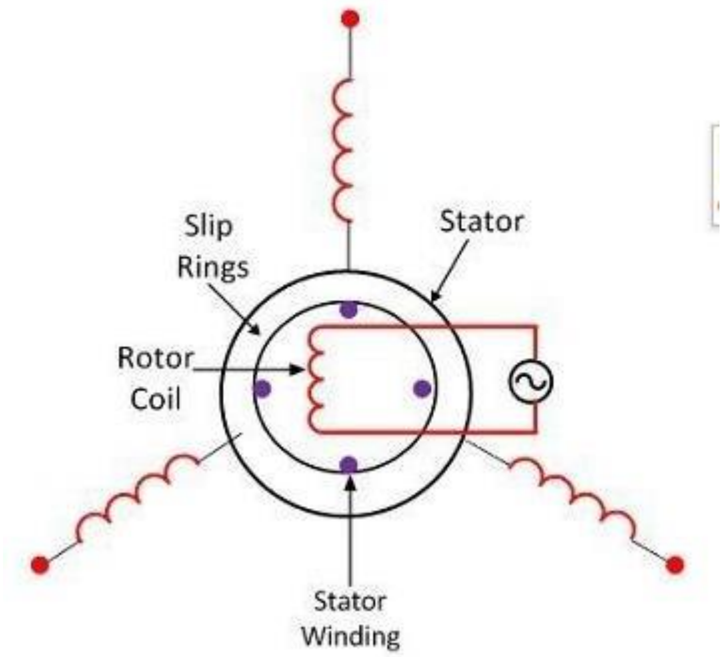
ControlTypeSynchrosSystem

Thecontrolssynchrosisusedforerrordetectioninpositionalcontrolsystems.Theirsystemsconsisttwo units. They are

1. SynchroTransmitter
2. Synchroreceiver

3. Thesynchroalwaysworkswiththesetwoparts.Thedetailexplanationofsynchrostransmitter and receiver is given below.

4. **SynchrosTransmitter–**Theirconstructionissimilartothethreephasealternator.Thestatorof the synchros is made of steel for reducing the iron losses. The stator is slotted for housing the three phase windings. The axis of the stator winding is kept 120º apart from each other.



Constructional Feature of Synchros Transmitter

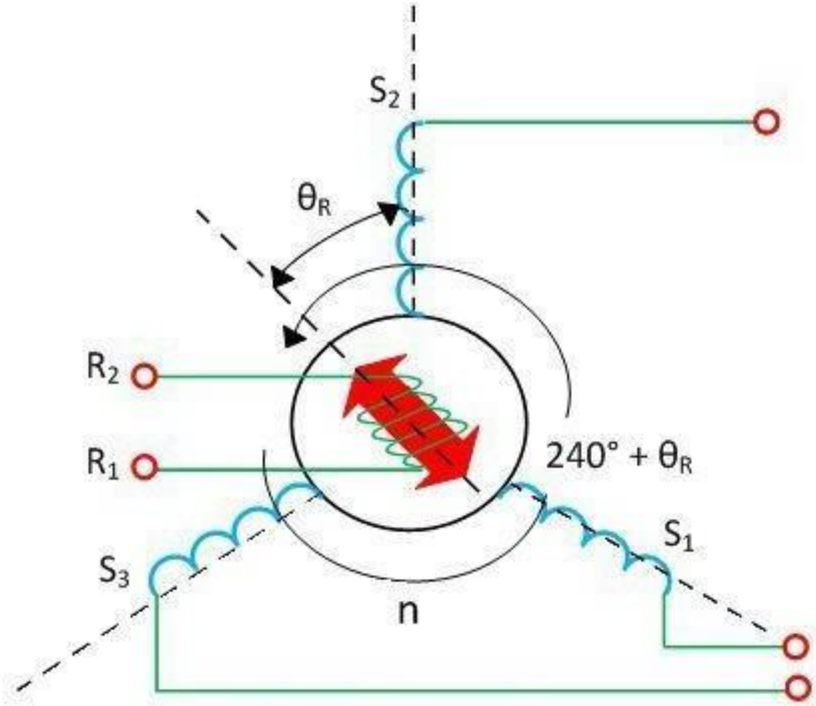
#

TheACvoltageisappliedtotherotorofthetransmitteranditisexpressedas

WhereV_r–r.ms.valueofrotorvoltage
ω_c–carrierfrequency

Thecoilsofthestatorwindingsareconnectedinstar.Therotorofthesynchrosisadumbbellin shape, and a concentric coil is wound on it. The AC voltage is applied to the rotor with the hel

constructionalfeatureofthesynchrosisshowninthefigurebelow.

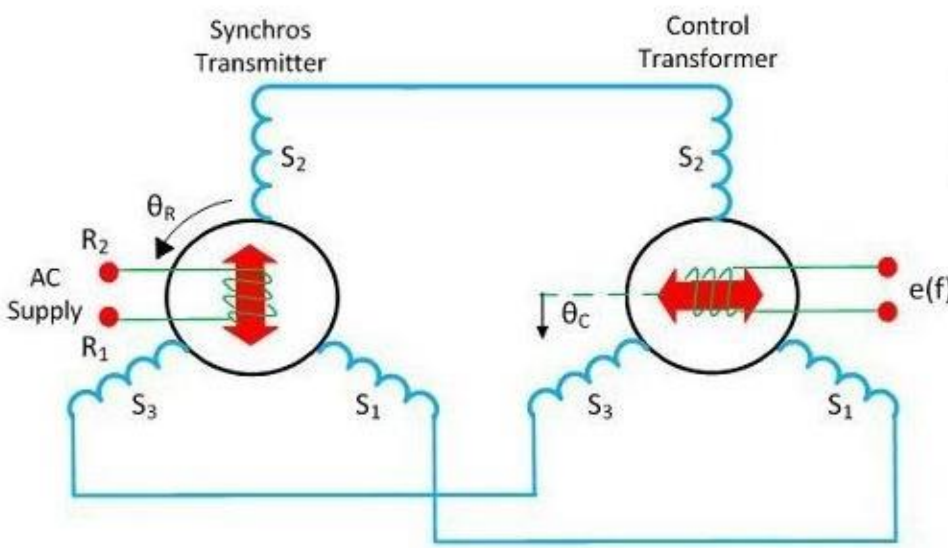


Sychro Transmitter

Considerthevoltageisappliedtotherotorofthetransmitterasshownin the figure

Sychro Transmitter

Consider the voltage is applied to the rotor of the transmitter as shown in the figure



Syncho Error Dectector

The voltage applied to the rotor induces the magnetizing current and an alternating flux along its axis.Thevoltageisinducedinthestatorwindingbecauseofthemutualinductionbetweentherotor and stator flux. The flux linked in the stator winding is equal to the cosine of the angle between the rotor and stator. The voltage is induced in the stator winding.

Let V_{s1},V_{s2},V_{s3}bethevoltagesgeneratedinthestatorwindingsS_1,S_2,andS_3respectively.Thefigure below shows the rotor position of the synchro transmitter. The rotor axis makes an angle

$$V_{s1n} = kV_r Sin\omega_c t cos(\theta_R + 120°)$$

$$V_{s2n} = kV_r Sin\omega_c t cos\theta_R$$

$$V_{s1n} = kV_r Sin\omega_c t cos(\theta_R + 240°)$$

$\theta_r$concerningthestatorwindingsS_2.

$$V_{s1s2} = V_{s1n} - V_{s2n}$$

$$V_{s1s2} = \sqrt{3}kV_r Sin(\theta_R + 240°)Sin\omega_c t$$

$$V_{s3s2} = V_{s2n} - V_{s3n}$$

$$V_{s1s2} = \sqrt{3}kV_r Sin(\theta_R + 120°)Sin\omega_c t$$

$$V_{s3s1} = V_{s3n} - V_{s1n}$$

$$V_{s3s1} = kV_r Sin\omega_c t Sin\theta_R$$

Thethreeterminalsofthestatorwindings are

Thevariationinthestatorterminalaxisconcerningtherotorisshowninthefigurebelow.

$$e(t) = k'V_r Cos(90° - \theta_R + \theta_C)Sin\omega_c t$$

$$e(t) = k'V_r Sin(\theta_R - \theta_C)Sin\omega_c t$$

Whentherotoranglebecomeszero,themaximumcurrentisproduced inthestatorwindingsS_2. The zero position of the rotor is used as a reference for determining the rotor angular position.

Theoutputofthetransmitterisgiventostatorwindingofthecontroltransformerwhichis shown in the above figure.

Thecurrentofthesameandmagnitudeflowthroughthetransmitterandcontroltransformerofthe synchros. Because of the circulating current, the flux is established between the air gap flux of the control transformer.

The flux axis of the control transformer and the transmitter is aligned in the same position. The voltagegeneratesbytherotorofcontroltransformerisequaltothecosineoftheanglebetweenthe rotors of the transmitter and the controller. The voltage is given as $e(t) = k'V_r Cos\emptyset Sin\omega_c t$

The voltage applied to the rotor induces the magnetizing current and an alternating flux along its axis.Thevoltageisinducedinthestatorwindingbecauseofthemutualinductionbetweentherotor and stator flux. The flux linked in the stator winding is equal to the cosine of the angle between the rotor and stator. The voltage is induced in the stator winding.

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$$V_{s1n} = kV_r Sin\omega_c t cos(\theta_R + 120°)$$

$$V_{s2n} = kV_r Sin\omega_c t cos\theta_R$$

$$V_{s1n} = kV_r Sin\omega_c t cos(\theta_R + 240°)$$

$\theta_r$concerningthestatorwindingsS_2.

$$V_{s1s2} = V_{s1n} - V_{s2n}$$

$$V_{s1s2} = \sqrt{3}kV_r Sin(\theta_R + 240°)Sin\omega_c t$$

$$V_{s3s2} = V_{s2n} - V_{s3n}$$

$$V_{s1s2} = \sqrt{3}kV_r Sin(\theta_R + 120°)Sin\omega_c t$$

$$V_{s3s1} = V_{s3n} - V_{s1n}$$

$$V_{s3s1} = kV_r Sin\omega_c t Sin\theta_R$$

5. Thethreeterminalsofthestatorwindings are

Thevariationinthestatorterminalaxisconcerningtherotorisshowninthefigurebelow.

$$e(t) = k'V_r Cos(90° - \theta_R + \theta_C)Sin\omega_c t$$

$$e(t) = k'V_r Sin(\theta_R - \theta_C)Sin\omega_c t$$

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$$e(t) = k'V_r Cos\emptyset Sin\omega_c t$$

Whereϕ–angulardisplacementbetweentherotoraxesoftransmitterandcontroller.
Whereϕ–angulardisplacementbetweentherotoraxesoftransmitterandcontroller.
Φ–90ºtheaxisbetweentherotoroftransmitterandcontroltransformerisperpendicularto each other.
The above figure shows the zero position of the rotor of transmitter and receiver.

Considerthepositionoftherotorandthetransmitterischanginginthesamedirection.Anangle $\theta_R$deflectstherotorofthetransmitterandthatofthecontroltransformeriskept θ_C.Thetotalangular separationbetweentherotorsisΦ= $(90º–\theta_R+ \theta_C)$

TherotorterminalvoltageoftheSynchrotransformerisgivenas

$$e(t) = k'V_r Cos(90° - \theta_R + \theta_C) Sin\omega_c t$$

$$e(t) = k'V_r Sin(\theta_R - \theta_C) Sin\omega_c t$$

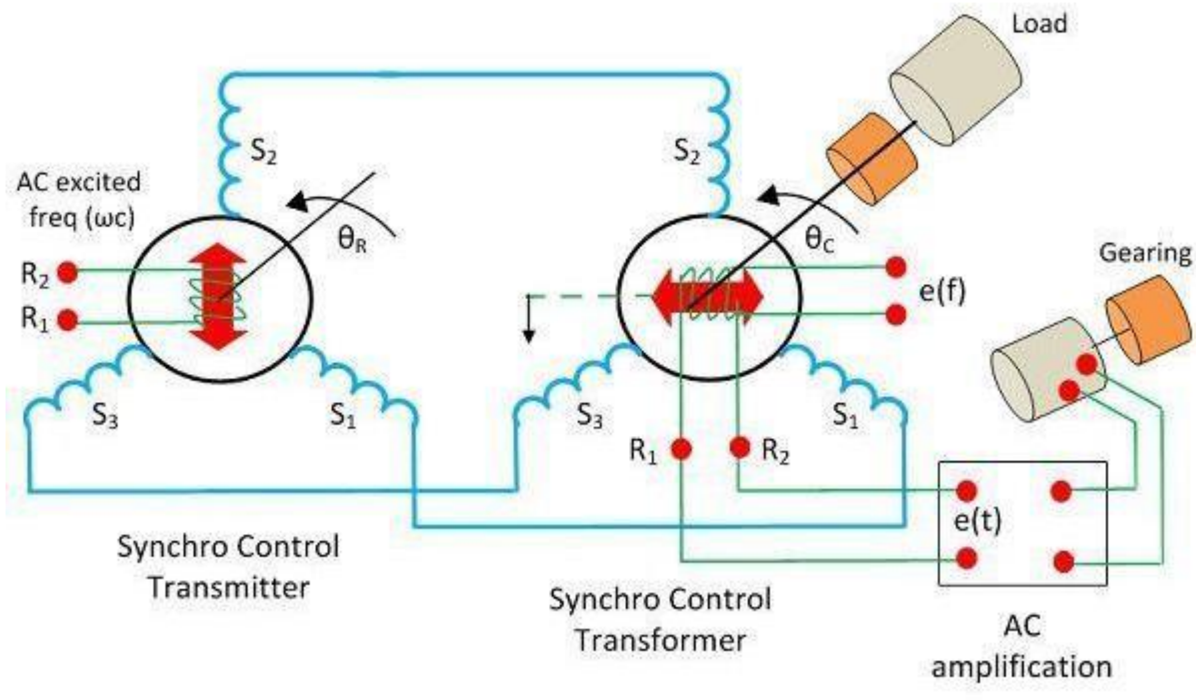
Thesmallangulardisplacementbetweentheirrotor positionisgivenas

$Sin (\theta_R- \theta_C)= (\theta_R-\theta_C)$

Onsubstitutingthevalueofangulardisplacementinequation(1)weget

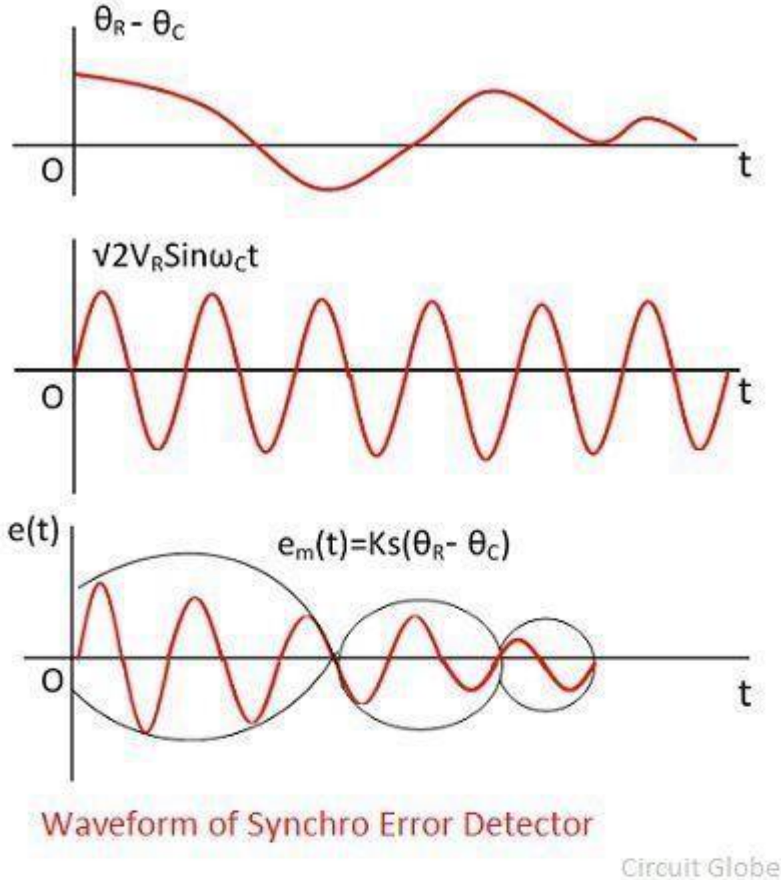
$$e(t) = k'V_r Cos\emptyset Sin\omega_c t$$

The synchro transmitter and the control transformer together used for detecting the error. The voltage equationshownaboveisequaltotheshaftpositionofthe rotorsofcontroltransformerandtransmitter.



Positional Control System

Theerrorsignalisappliedtothedifferentialamplifierwhichgivesinputtotheservomotor.Thegearof the servo motor rotates the rotor of the control transformer



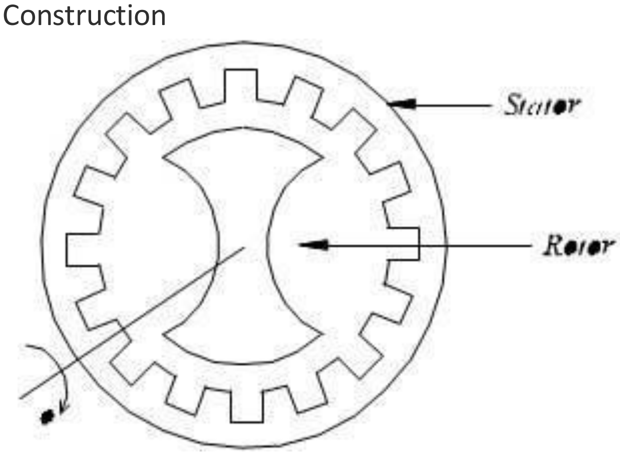
The figure above shows the output of the synchro error detector which is a modulated signal. The modulatingwaveaboveshownthemisalignmentbetweentherotorpositionandthecarrierwave.

$$e(t) = (\theta_R - \theta_C)$$

Where$K_s$istheerrordetector.

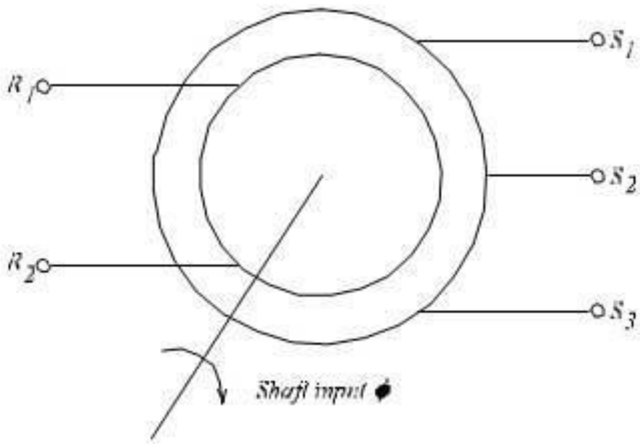
SYNCHROCONTROLTRANSFORMER

Construction



Figure–4aConstructional Features

The constructionalfeatures of **synchro control transformer** are similarto that of **Synchro Transmitter**, excepttheshapeof rotor. Therotorof the control transformer ismadecylindrical so thattheair gap is practically uniform. This feature of the control transformer minimizes the changes in the rotor impedancewiththerotationoftheshaft.Theconstructionalfeatures,electricalcircuitandaschematic symbol of control transformer are shown in figure 4.



Figure–4bSchematicSymbolof**synchrocontrol transformer**

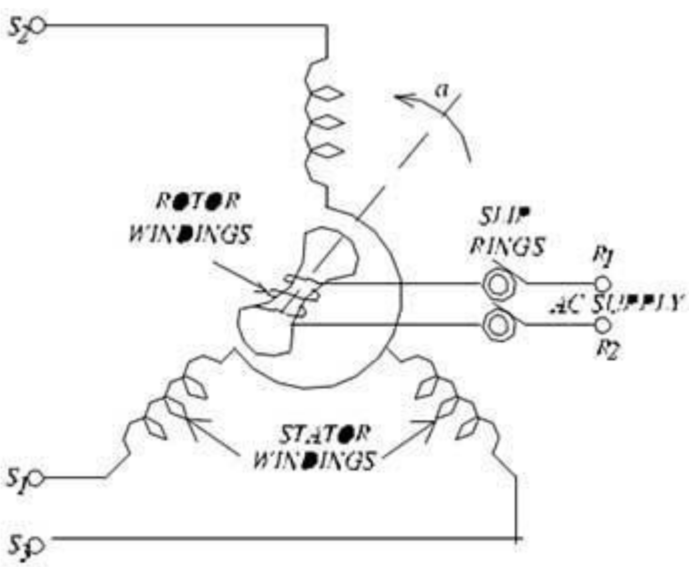


Figure –4cElectricalCircuitof **synchrocontrol transformer**

Working

The generated emf of the **Synchro Transmitter** is applied as input to the stator coils of control transformer.Therotorshaftisconnectedtotheloadwhosepositionhastobemaintainedatthedesired value. Depending on the current position of the rotor and the applied emf on the stator, an emf is induced on the rotor winding. This emf can be measured and used to drive a motor so that the position of the load is corrected

3.3 ElectricalTachometer

Definition: The tachometer use for measuring the rotational speed or angular velocity of the machine whichiscoupledtoit.Itworksontheprincipleofrelativemotionbetweenthemagneticfieldandshaft of the coupled device. The relative motion induces the EMF in the coil which is placed between the constant magnetic field of the permanent magnet. The develops EMF is directly proportional to the speed of the shaft.

Mechanicalandelectricalarethetwotypesofthetachometer.Themechanicaltachometermeasures the speed of shaft regarding revolution per minutes.

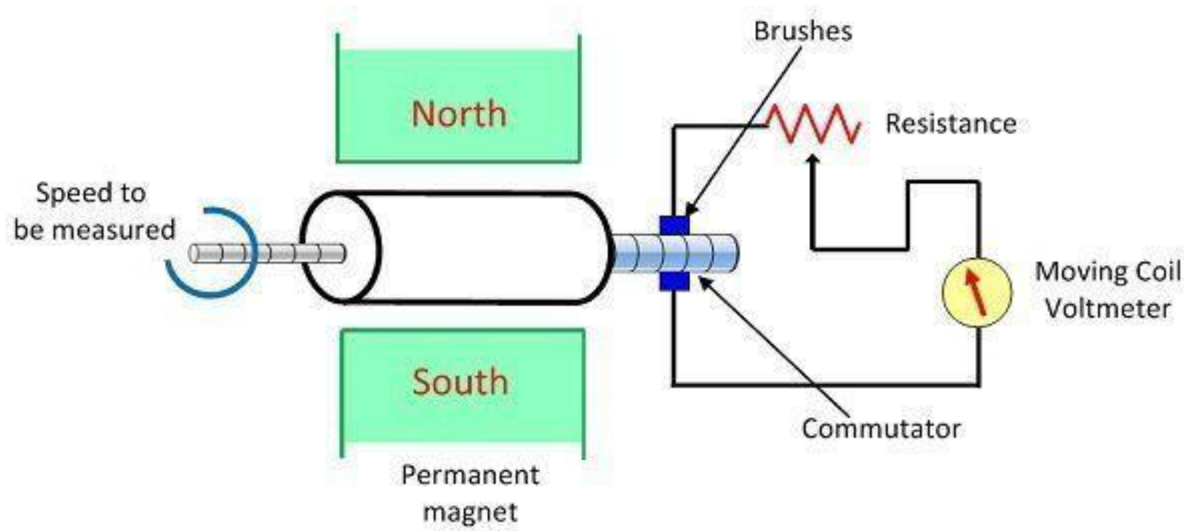
The electrical tachometer converts the angular velocity into an electrical voltage. The electrical tachometerhasmoreadvantagesoverthemechanicaltachometer.Thusitismostlyusedformeasuring the rotational speed of the shaft. Depends on the natures of the induced voltage the electrical tachometer is categorized into two types.

- ACTachometerGenerator
- DCTachometerGenerator

DCTachometerGenerator

Permanentmagnet,armature,commutator,brushes,variableresistor,andthemovingcoilvoltmeter are the main parts of the DC tachometer generator. The machine whose speed is to be measured is coupled with the shaft of the DC tachometer generator.

TheDCtachometerworksontheprinciplethatwhentheclosedconductormovesinthemagneticfield, EMF induces in the conductor. The magnitude of the inducesemf depends on the flux link with the conductor and the speed of the shaft.



DC Tachometer Generator

Circuit Globe The armature of the DC generator revolves between the constant field of the permanent magnet. The rotationinducestheemfinthecoil.Themagnitudeoftheinducedemfisproportionaltotheshaft speed.

The commutatorconvertsthealternatingcurrentofthearmaturecoiltothedirectcurrentwiththe help ofthebrushes.Themovingcoilvoltmetermeasurestheinducedemf.Thepolarityoftheinducesvoltage determines the direction of motion of the shaft. The resistance is connected in series with thevoltmeterforcontrollingtheheavycurrentofthearmature.

Theemfinducesinthedctachometergeneratorisgiven as

$$E = \frac{\emptyset P N}{60} \times \frac{z}{a}$$

Where,E–generatedvoltage
Φ–fluxperpolesinWeber P-
number of poles
N–speedinrevolutionperminutes
Z–thenumberoftheconductorinarmaturewindings.
a–numberoftheparallelpathinthearmature windings.

$$E \, \alpha \, N$$

$$E = KN$$

$$K = Constant = \frac{\emptyset P}{60} \times \frac{z}{a}$$

AdvantagesoftheDCGenerator

ThefollowingaretheadvantagesoftheDCTachometer.

- Thepolarityoftheinducesvoltagesindicatesthedirectionofrotationoftheshaft.
- TheconventionalDCtypevoltmeterisusedformeasuringtheinducesvoltage.

DisadvantagesofDCGenerator

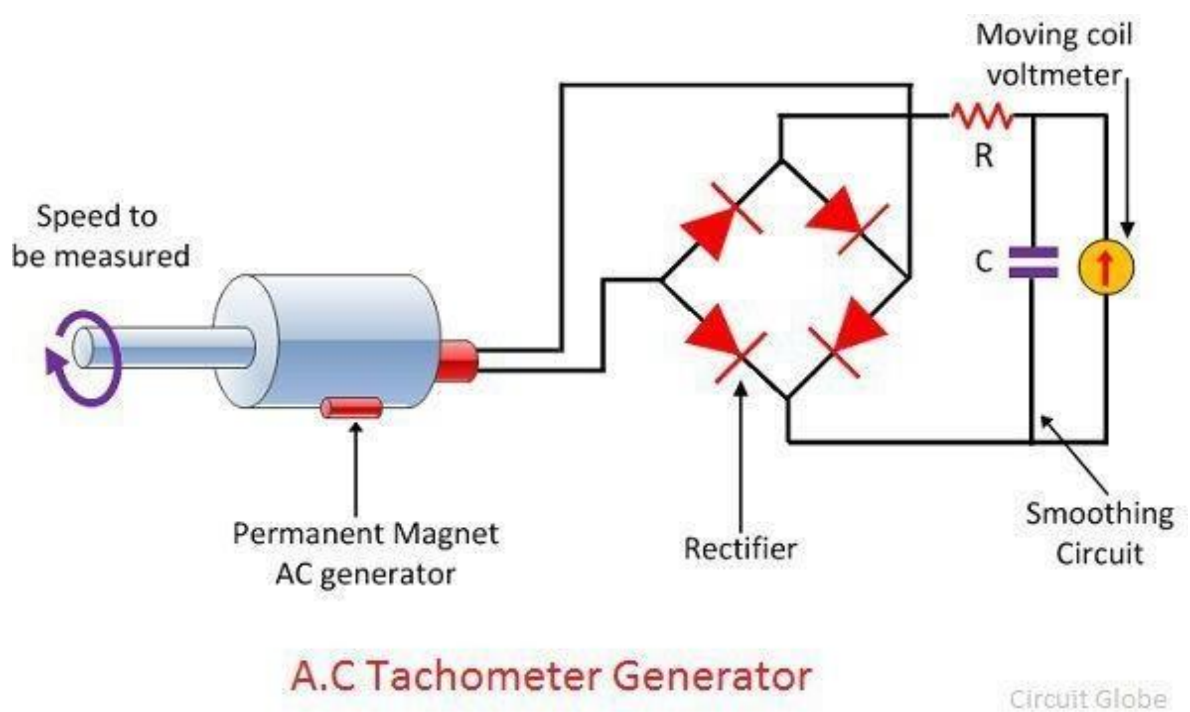
- Thecommutatorandbrushesrequiretheperiodicmaintenance.
- The output resistance of the DC tachometer is kept high as compared to the input resistance.Ifthelargecurrentisinducedinthearmatureconductor,theconstantfieldof the permanent magnet will be distorted.

ACTachometerGenerator

TheDCtachometergeneratorusesthe commutator andbrusheswhichhavemanydisadvantages.The AC tachometer generator designs for reducing the problems. The AC tachometer has stationary armatureandrotatingmagneticfield.Thus,thecommutatorandbrushesareabsentinACtachometer generator.

TherotatingmagneticfieldinducestheEMFinthestationarycoilofthe stator.Theamplitudeand frequency of the induced emf are equivalent to the speed of the shaft. Thus, either amplitude or frequency is used for measuring the angular velocity.

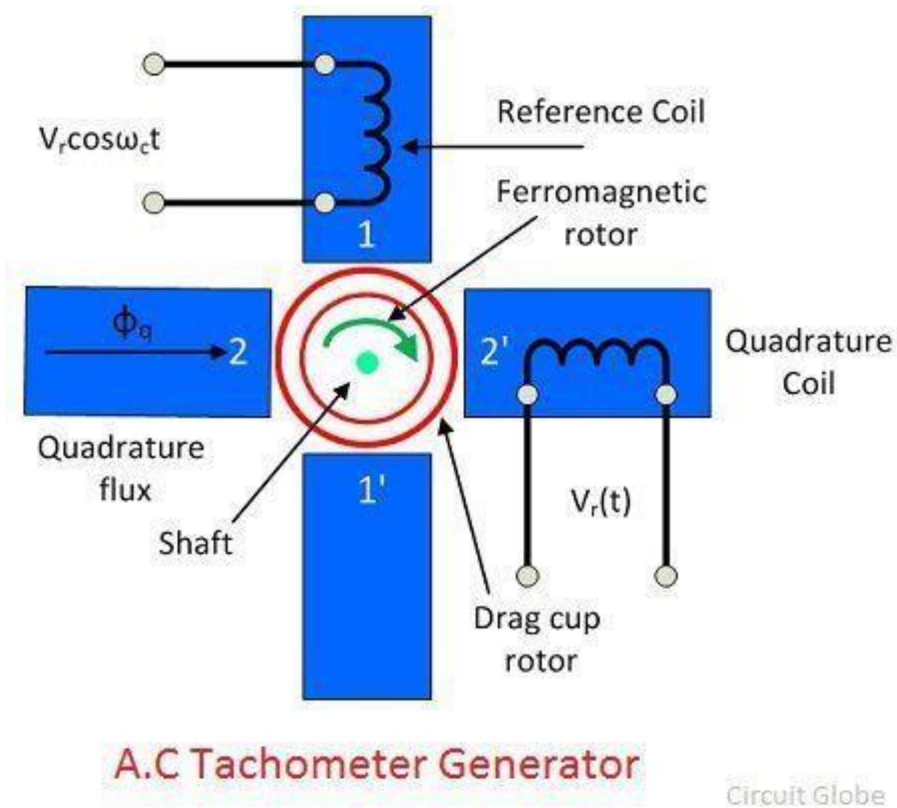
Thebelowmentioncircuitisusedformeasuringthespeedoftherotorbyconsideringtheamplitudeof the induced voltage. The induces voltages are rectified and then passes to the capacitor filter for smoothening the ripples of rectified voltages.



Drag Cup

RotorAC Generator

ThedragcuptypeA.Ctachometerisshowninthefigurebelow.



The stator of the generator consiststwowindings,i.e.,thereferenceandquadraturewinding.Boththewindingsaremounted90° apart from each other. The rotor of the tachometer is made with thin aluminium cup, and it is placed between the field structure.

Therotorismadeofthehighlyinductivematerialwhichhaslowinertia.The inputisprovidedtothe reference winding, and the output is obtained from the quadrature winding. The rotation of rotor between the magnetic field induces the voltage in the sensing winding. The induces voltage is proportional to the speed of the rotation.

Advantages

- ThedragcupTachogeneratorgeneratestheripplefreeoutput voltage.
- Thecostofthegeneratorisalsovery less.

Disadvantage

Thenonlinearrelationshipobtainsbetweentheoutputvoltageandinputspeedwhentherotorrotates at high speed.

ThefollowingaretheadvantagesoftheDCTachometer.

- Thepolarityoftheinducesvoltagesindicatesthedirectionofrotationoftheshaft.
- TheconventionalDCtypevoltmeterisusedformeasuringtheinducesvoltage.

DisadvantagesofDCGenerator

- Thecommutatorandbrushesrequiretheperiodicmaintenance.
- The output resistance of the DC tachometer is kept high as compared to the input resistance.Ifthelargecurrentisinducedinthearmatureconductor,theconstantfieldof the permanent magnet will be distorted.

ServoMotor

Servo Motor are also called Control motors. They are used in feedback control systems as output actuatorsanddoesnotuseforcontinuousenergyconversion.TheprincipleoftheServomotorissimilar to that of the other electromagnetic motor, but the construction and theoperation are different. Their power rating varies from a fraction of a watt to a few hundred watts.

Therotorinertiaofthemotorsislowandhaveahighspeedofresponse.TherotoroftheMotorhasthe long length and smaller diameter. They operate at very low speed and sometimes even at the zero speed.Theservo motor is widely used in radar and computers, robot, machine tool, tracking and guidance systems, processing controlling, etc.

ApplicationsoftheServoMotor

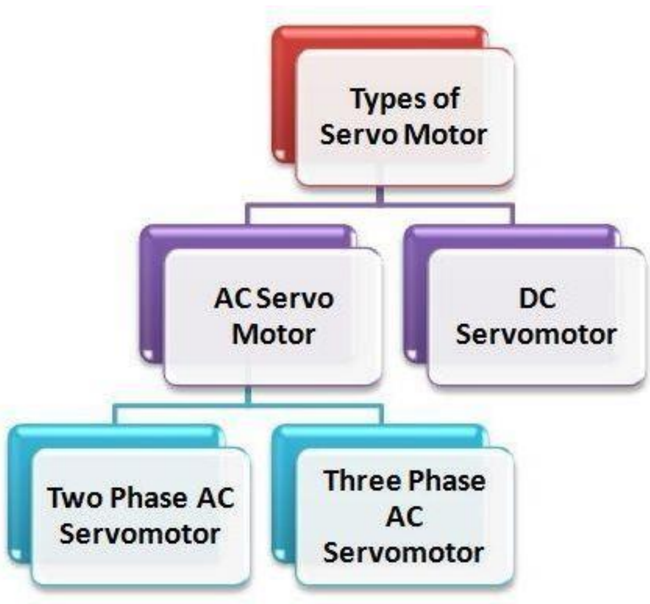
Thepowerratingoftheservomotormayvaryfromthefractionofwattstofewhundredsofwatts.The rotor of servo motor have low inertia strength, and therefore they have a high speed of inertia. The Applications of the Servomotor are as follows:-

- TheyareusedinRadarsystemandprocess controller.
- Servomotorsareusedincomputersandrobotics.
- Theyarealsousedinmachinetools.
- Trackingandguidancesystems.

ClassificationofServoMotor

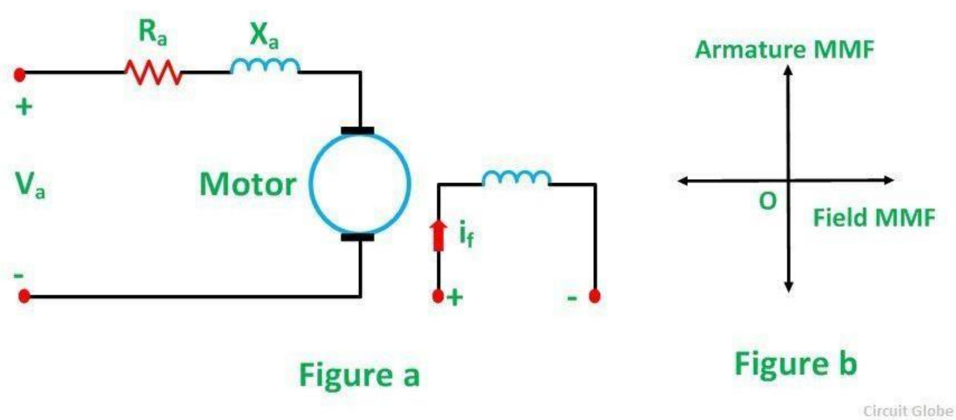
TheyareclassifiedasACandDCServoMotor.TheACservomotorisfurtherdividedintotwotypes.

- TwoPhaseACServoMotor
- ThreePhaseACServoMotor



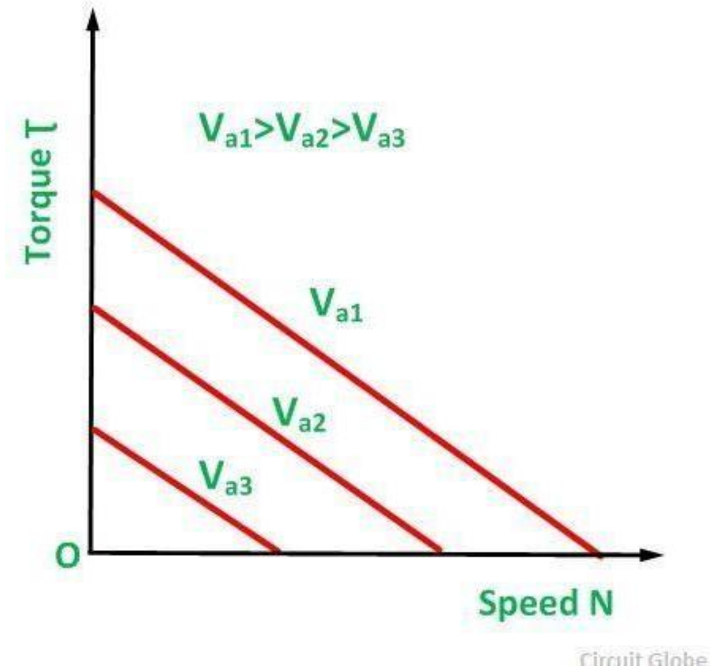
3.3DC servomotor

DC Servo Motors are separately excited DC motor or permanent magnet DC motors. The figure (a) showstheconnectionofSeparatelyExcitedDCServomotorandthefigure(b)showsthearmatureMMF and the excitation field MMF in quadrature in a DC machine.



This provides a fast torque response because torque and flux are decoupled. Therefore, a small change inthearmaturevoltageorcurrentbringsasignificantshiftinthepositionorspeedoftherotor.Most of the high power servo motors are mainly DC.

TheTorque-SpeedCharacteristicsoftheMotorisshownbelow.



Asfromtheabovecharacteristics,itisseenthattheslopeisnegative.Thus,anegativeslopeprovides viscous damping for the servo drive system.

3.4ACServoMotor

ACServo Motors
AC servo motors are basically two-phase squirrel cage induction motors and are used for low power applications.Nowadays,threephasesquirrelcageinductionmotorshavebeenmodifiedsuchthatthey can be used in high power servo systems.

Themaindifferencebetweenastandardsplit-phaseinductionmotorandACmotoristhatthesquirrel cage rotor of a servo motor has made with thinner conducting bars, so that the motor resistance is higher.

ACServoMotor

Basedontheconstructionthereraretwodistincttypesof ACservomotors,theyaresynchronoustype AC servo motor and induction type AC servo motor.

Synchronous-type AC servo motor consist of stator and rotor. The stator consists of a cylindrical frame andstatorcore.Thearmaturecoilwoundaroundthestatorcoreandthecoilendisconnectedtowitha lead wire through which current is provided to the motor.

Therotorconsistsofapermanentmagnetandhencetheydonotrelyon ACinductiontyperotorthat has current induced into it. And hence these are also called as brushless servo motors because of structural characteristics.

Synchronous-typeACservo motor

When the stator field is excited, the rotor follows the rotating magnetic field of the stator at the synchronousspeed.Ifthestatorfieldstops,therotoralsostops.Withthispermanentmagnetrotor,no rotor current is needed and hence less heat is produced.

Also, these motors have high efficiency due to the absence of rotor current. In order to know the positionofrotorwithrespecttostator,anencoderisplacedontherotorandit actsasafeedbacktothe motor controller.

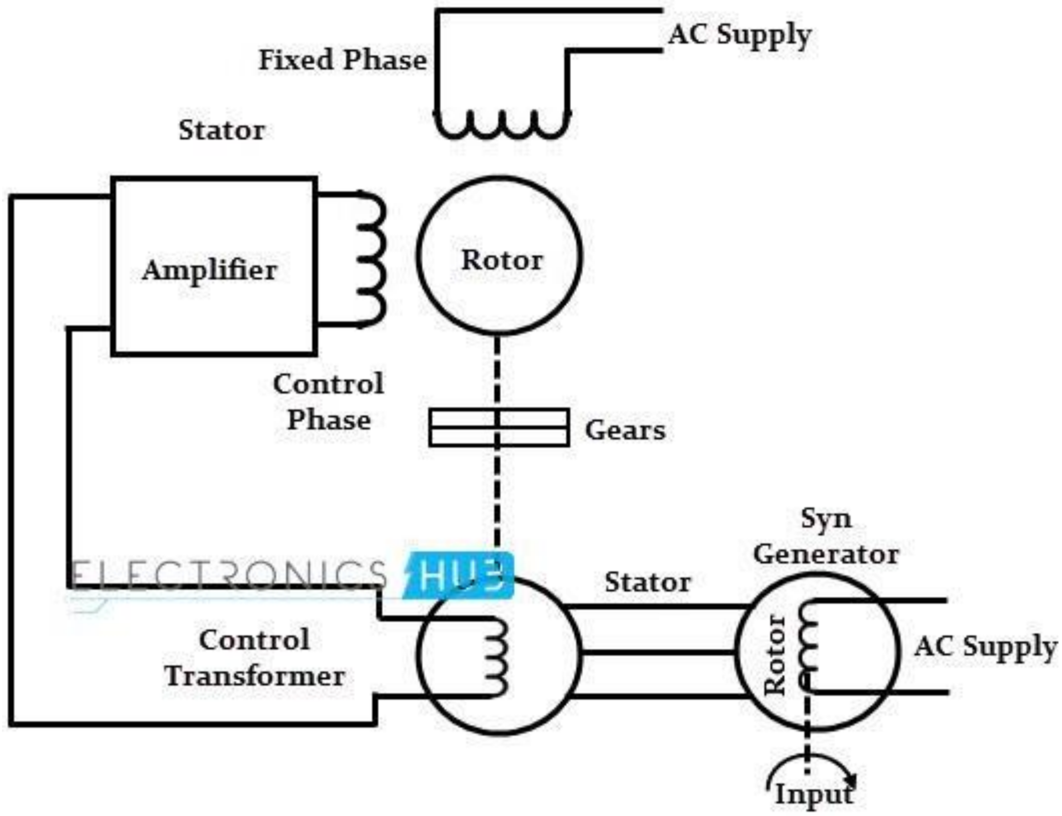
The **induction-type AC servo moto**r structure is identical with that of general motor. In this motor, statorconsistsofstatorcore,armaturewindingandleadwire,whilerotorconsistsofshaftandtherotor core that built with a conductor as similar to squirrel cage rotor.

induction-typeACservomotor

Theworkingprincipleofthisservomotorissimilartothenormalinductionmotor.Againthecontroller must know the exact position of the rotor using encoder for precise speed and position control.

WorkingPrincipleofACServoMotor

TheschematicdiagramofservosystemforACtwo-phaseinductionmotorisshowninthefigurebelow. In this,thereferenceinput atwhich themotor shafthastomaintainatacertainposition isgivento the rotor of synchro generator as mechanical input theta. This rotor is connected to the electrical input at rated voltage at a fixed frequency.



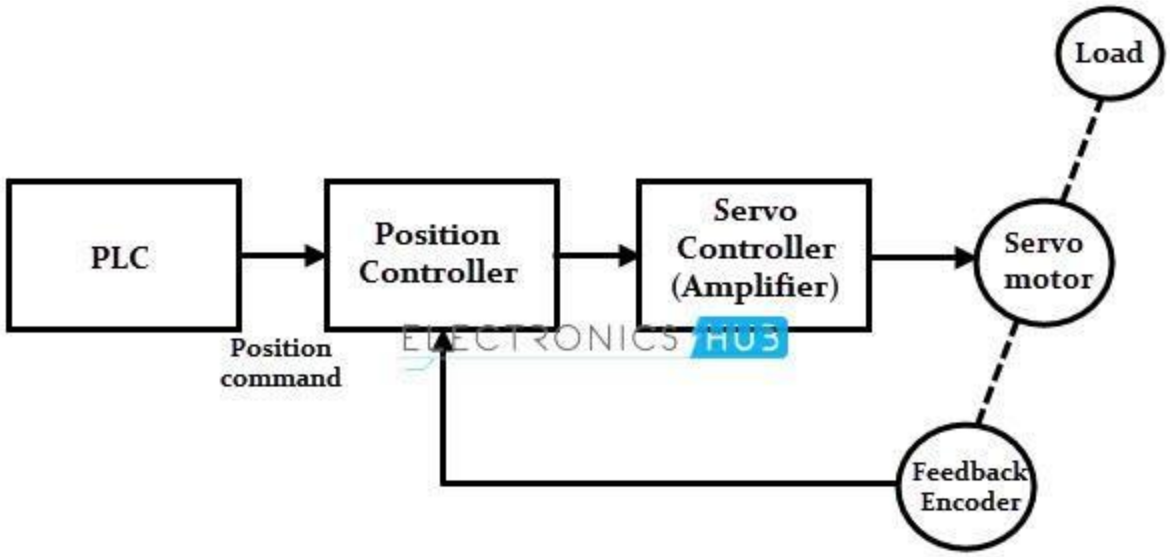
The three stator terminals of a synchro generator are connected correspondingly to the terminals of controltransformer.Theangularpositionofthetwo-phasemotoristransmittedtotherotorofcontrol transformer through gear train arrangement and it represents the control condition alpha.

Initially,thereexistadifferencebetweenthesynchrogeneratorshaftpositionandcontroltransformer shaftposition.Thiserrorisreflectedasthevoltageacrossthecontroltransformer.Thiserrorvoltageis applied to the servo amplifier and then to the control phase of the motor.

Withthecontrolvoltage,therotorofthemotorrotatesinrequireddirectiontilltheerrorbecomeszero. This is how the desired shaft position is ensured in AC servo motors.

Alternatively,modernACservodrivesareembeddedcontrollerslikePLCs,microprocessorsand microcontrollerstoachievevariablefrequencyandvariablevoltageinordertodrivethemotor.

Mostly,pulsewidthmodulationandProportional-Integral-Derivative(PID)techniquesareusedto control the desired frequency and voltage. The block diagram of AC servo motor system using programmable logic controllers, position and servo controllers is given below.



DifferencebetweentheDCandACServoMotors

DCSERVOMOTOR	**ACSERVOMOTOR**
Itdelivershighpoweroutput	Deliverslowoutputofabout 0.5Wto100W
Ithasmorestabilityproblems	Ithasless stableproblems
Itrequiresfrequentmaintenancedueto thepresenceofcommutator	Itrequireslessmaintenanceduetotheabsenceofcommutator
Itprovideshighefficiency	TheefficiencyofACservomotorislessandisabout5to20%
The life of DC servo motor depends on the on brush life	ThelifeofACservomotordependsonbearinglife life
Itincludespermanentmagnetinits construction	ThesynchronoustypeACservomotorusespermanentmagnetwhil
Thesemotorsareusedforhighpower applications	Thesemotorsareusedforlowpowerapplications

Shortquestions

1. defineGyroscope

Ans-itisaninstrumentusedinspaceshipandaircrafts.the inputistheangularvelocityandtheoutputis the angular displacement

2. Definetachometer?

Ans-itisanminiaturelowvoltagegeneratorwheretheoutputvoltageofgeneratorisgivenbyE_f=kfw

3. definesynchro?

Ans-itisanelectromechanicaldevicethatproduceanoutputvoltagedependingontheangularposition of rotor and not on rotor speed

Longquestions

1. Explainsynchrotransmitter?

2. Explainsynchroreceiver?

Explainwithdiagramdcandacservo motor?

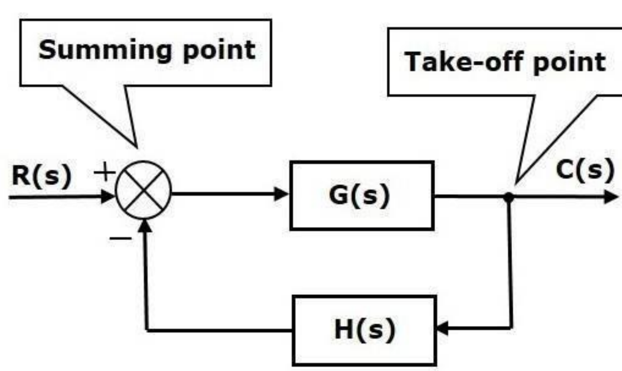
Ch-4

Blockdiagramalgrbraandsignalflow graph

Blockdiagramsconsistofasingleblockoracombinationofblocks.Theseareusedtorepresent the control systems in pictorial form.

BasicElementsofBlockDiagram

The basic elements of a block diagram are a block, the summing point and the take-off point. Let us consider the block diagram of a closed loop control system as shown in the following figure to identify these elements.

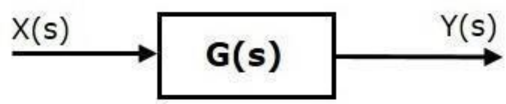


TheaboveblockdiagramconsistsoftwoblockshavingtransferfunctionsG(s)andH(s).Itis also having one summing point and one take-off point. Arrows indicate the direction of the flow of signals. Let us now discuss these elements one by one.

Block

Thetransferfunctionofacomponentisrepresentedbyablock.Blockhassingleinputandsingle output.

ThefollowingfigureshowsablockhavinginputX(s),outputY(s)andthetransferfunctionG(s).



TransferFunction,$G(s)=Y(s)X(s)$

$\Rightarrow Y(s)=G(s)X(s)$

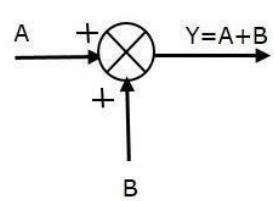
Outputoftheblockisobtainedbymultiplyingtransferfunctionoftheblockwithinput.

SummingPoint

The summing point is represented with a circle having cross (X) inside it. It has two or more inputs and single output. It produces the algebraic sum of the inputs. It also performs the summationorsubtractionorcombinationofsummationandsubtractionoftheinputsbasedon the polarity of the inputs. Let us see these three operations one by one.

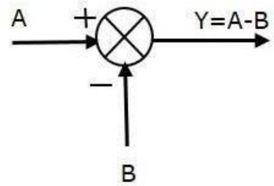
The following figure shows the summing point with two inputs (A, B) and one output (Y). Here, the inputs A and B havea positive sign. So,the summing pointproducesthe output, Yas **sum of A and B.**

i.e.,Y= A+ B.



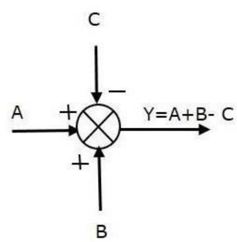
The following figure shows the summing point with two inputs (A, B) andone output (Y). Here, theinputsAandBarehavingoppositesigns,i.e.,A ishavingpositivesignandBishavingnegative sign. So, the summing point produces the output **Y** as the **difference of A and B**.

Y =A+(-B)=A-B.



The following figure shows the summing point with three inputs (A, B, C) and one output (Y). Here, the inputs A and B are having positive signs and C is having a negative sign. So, the summing point produces the output **Y** as

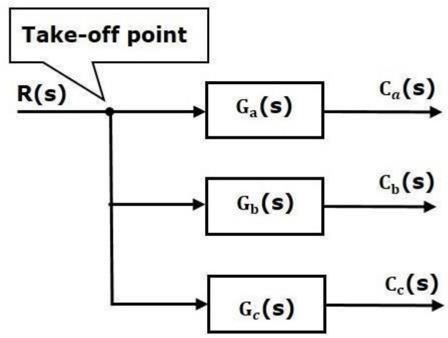
Y =A+ B+(−C)= A+B−C.



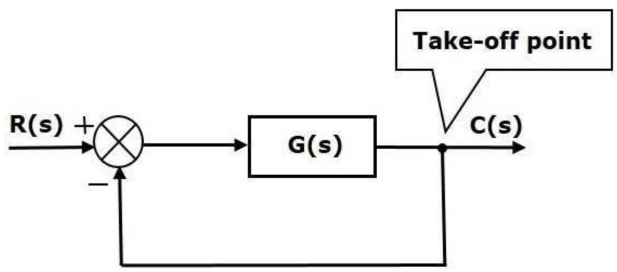
Take-off Point

Thetake-offpointisapointfromwhichthesameinputsignalcanbepassedthroughmorethan onebranch.That meanswith thehelp of take-offpoint, we can applythesame input toone or more blocks, summing points.

In the following figure, the take-off point is used to connect the same input, R(s) to two more blocks.



Inthefollowingfigure,thetake-offpointisusedtoconnecttheoutputC(s),asoneofthe inputs to the summing point.



ControlSystems- BlockDiagram Algebra

Blockdiagramalgebraisnothingbutthealgebrainvolvedwiththebasicelementsoftheblock diagram. This algebra deals with the pictorial representation of algebraic equations.

Block diagram of a closed loop system .

Fig2.3 shows an example of a block diagram of a closed system

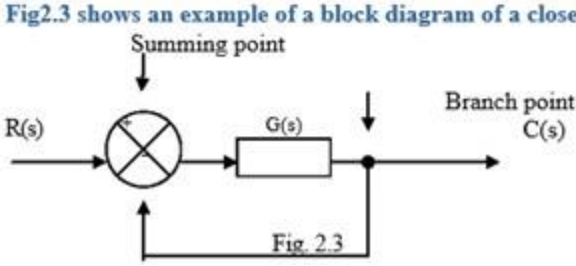
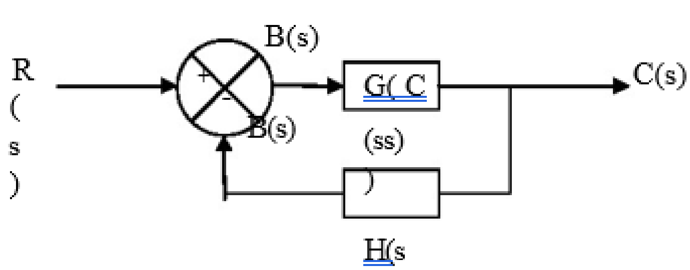


Fig. 2.3

The output C(s) is fed back to the summing point, where it is compared with reference input R(s). The closed loop nature is indicated in fig1.3. Any linear system may be represented by a block diagram consisting of blocks, summing pointsandbranchpoints.Abranchisthepointfromwhichtheoutputsignalfrom a block diagram goes concurrently to other blocks or summing points.

Whentheoutputisfedbacktothesumming pointforcomparisonwiththe input,itisnecessarytoconverttheformofoutputsignaltothatofheinputsignal. This conversion is followed by the feed back element whose transfer function is H(s) as shown in fig 1.4. Another important role of the feed back element is to modify the output before it is compared with the input.

input.



) Fig2.4

TheratioofthefeedbacksignalB(s)totheactuatingerrorsignalE(s)is called the open loop transfer function.

openlooptransferfunction=B(s)/E(s)=G(s)H(s)

TheratiooftheoutputC(s)totheactuatingerrorsignalE(s)iscalledthe feed forward transfer function .

Feedforwardtransferfunction=C(s)/E(s)=G(s)

If the feed back transfer function is unity, then the open loop and feed forwardtransferfunctionarethesame.ForthesystemshowninFig1.4,theoutput C(s) and input R(s) are related asfollows.

C(s)=G(s)E(s)

E(s)=R(s)-B(s)

= R(s) -H(s) C(s) but

B(s)= H(s)C(s)EliminatingE(s)from theseequations

C(s)=G(s)[R(s)-H(s) C(s)]

C(s)+G(s)[H(s)C(s)]=G(s)R(s)

C(s)[1+G(s)H(s)]= G(s)R(s)

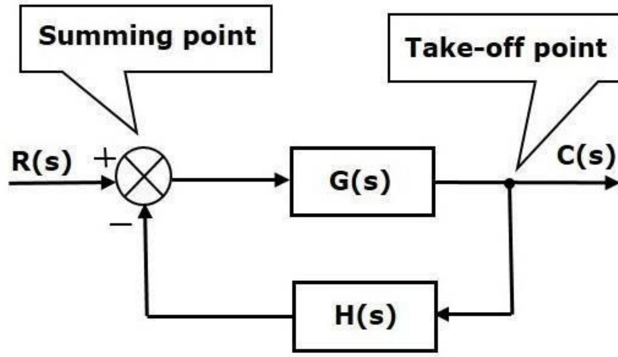
C(s)/r(s) G(s)/1+G(s)H(s)

C(s)/R(s) is called the closed loop transfer function.

Theoutputoftheclosedloopsystemclearlydependsonboththeclosed loop transfer function and the nature of the input. If the feed back signal is positive, then

C(s)/r(s) G(s)/1+G(s)H(s)

canonicalformofclosedloopblockdiagram



This fig shows a block diagram which consist of forward path having one block,feedback path havingone block,take off point and summingpoint.it representsa canonical form of close loop system.R(s) laplacetransform of reference input,c(s) is the la[lace transform of controlled output c(t),E(s) is the laplace transform of error signale(t),B(s) is the laplace transform of feedbacksignalb(t).c(s)istheequivqlentforwardpathtranferfunction,H(s)equivalentfeedback path transfer function.

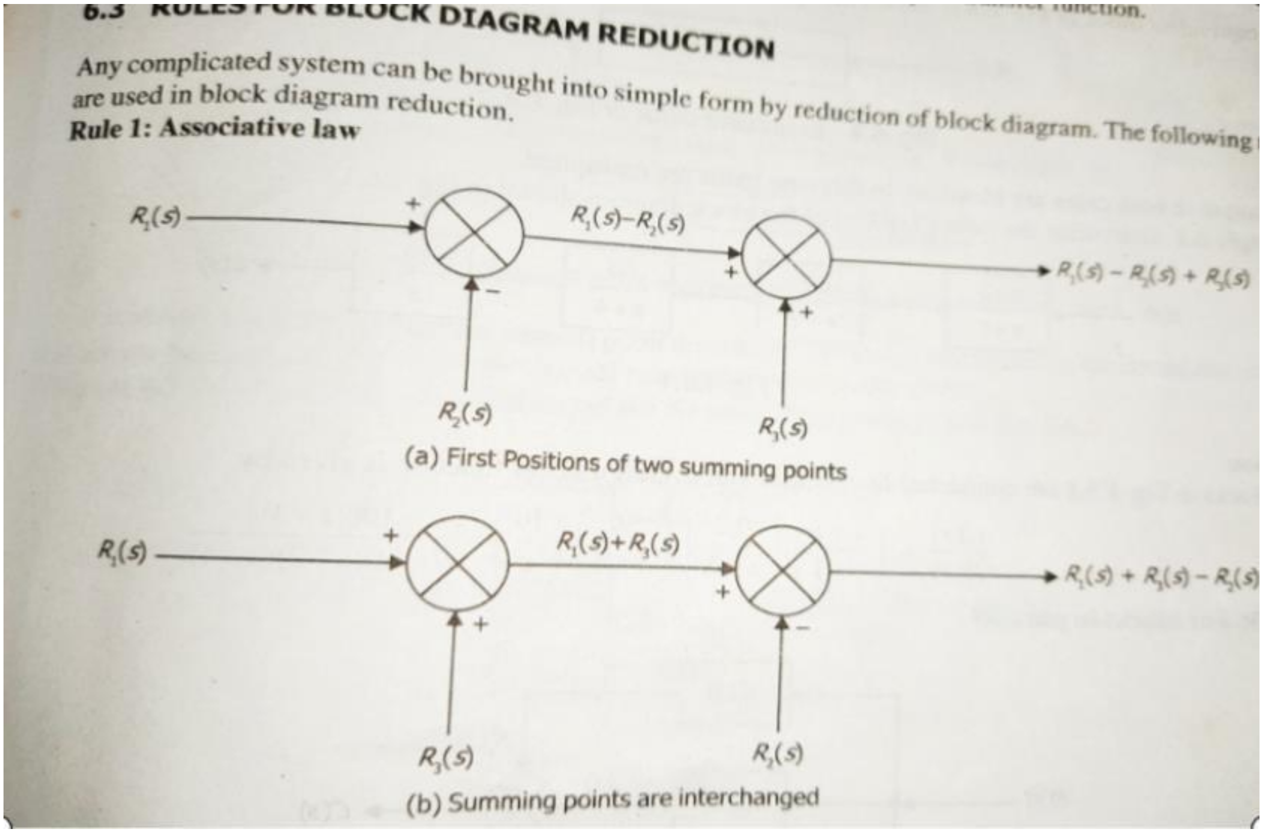
4.4:Procedureforreductionofblockdiagram

Rule 1:Associative law

In thebelow fig two summing points havebeen taken into account.in the1st case the output is $R_1(s)-R_2(s)+R_3(s)$.in fig b the position of summing point are interchanged.the output is $R_1(s)+R_2(s)-R_3(s)$

Fromfigaandb

$R_1(s)+R_3(s)-R_2(s)$



6.3 RULES FOR BLOCK DIAGRAM REDUCTION

Any complicated system can be brought into simple form by reduction of block diagram. The following are used in block diagram reduction.

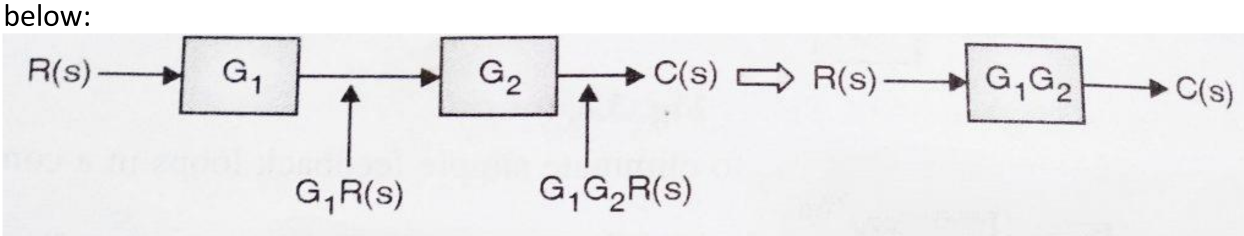
Rule 1: Associative law

(a) First Positions of two summing points

(b) Summing points are interchanged

Rule2:BlocksIn Series/cascade

Anyfinitespecificnumberofblocksarrangedinseriescanbecombinedtogetherbymultiplicationas shown below:



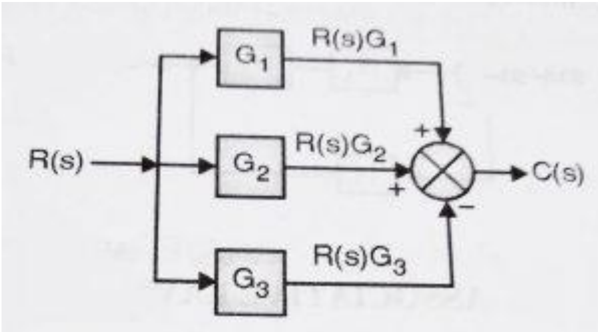
Theaboveblocksshowncanbecombinedtogetherandreplaced withsingleblockas

OutputC(s)=G1xG2xR(s)

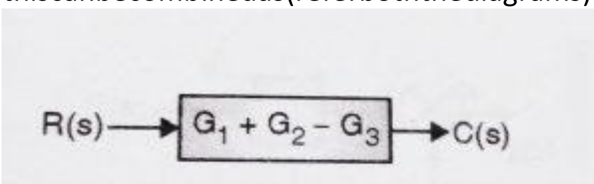
If there is a take-off point or summing point between the blocks, the blocks cannot be said to be in cascade/series.(thetake-off/summingpointhastobeshiftedbeforeoraftertheblockusinganother rule)

Rule3:BlocksIn Parallel

Whentheblocksareconnectedinparallelcombination,theygetaddedalgebraically(consideringthe sign of the signal)



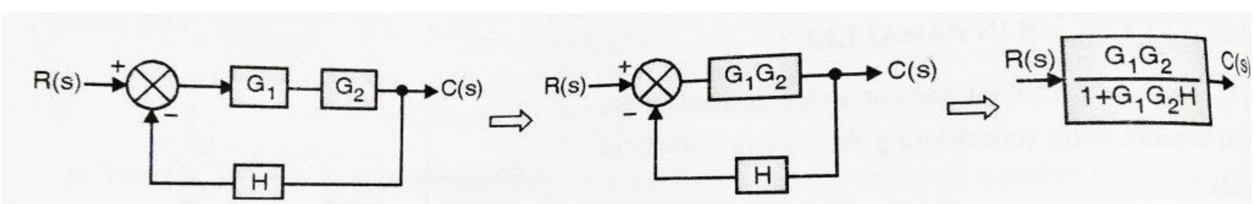
thiscanbecombinedas(referboththediagrams)



Theaboveblockscanbereplacedwithasingleblockas C(s) =
R(s)G1 + R(s)G2 – R(s)G3
C(s)= R(s)(G1+ G2 – G3)
If anysummingpoint/take-offpointispresent inbetweentheblocks,then thathastobeshiftedfirst.(in aparallelarrangement,thedirectionofsignalflow mustbeinthesamedirectionthrough alltheblocks)

Rule4:Eliminationoffeedback Loop

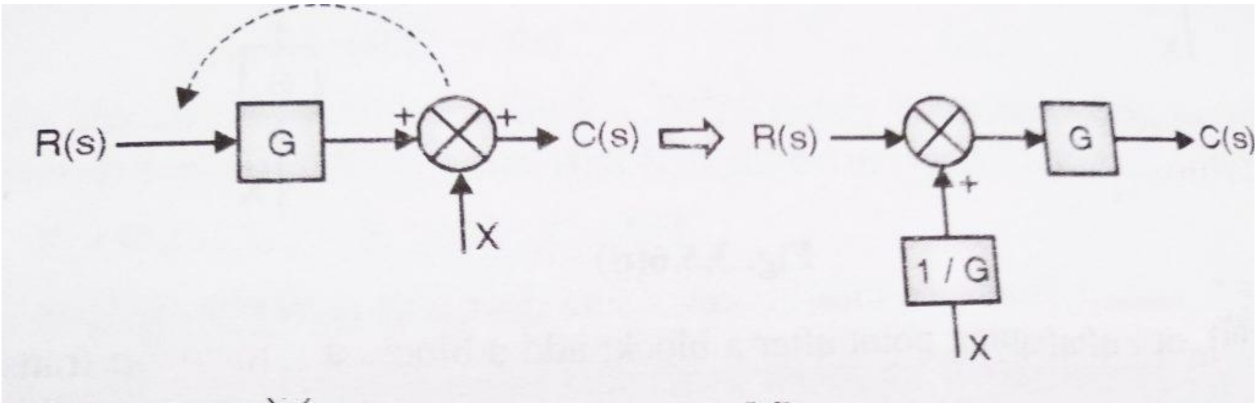
WecanuseClosedlooptransferfunctiontoeliminatethefeedbacklooppresent.(Alwaysrememberfor applying this method the direction offlow of signals should be in opposite direction, otherwise, if they are in the same direction, then we need to apply parallel reduction technique discussed above)



Nowconsidertheapplicationoftheabovethreerulestogetherand refertotheblockdiagram above.

Rule5:ShiftingofaSumming Pointbeforeablock

Whenweshiftthesummingpointbeforeablock,weneedtodothetransformation inordertoachieve the same result. Please refer to the diagram below :

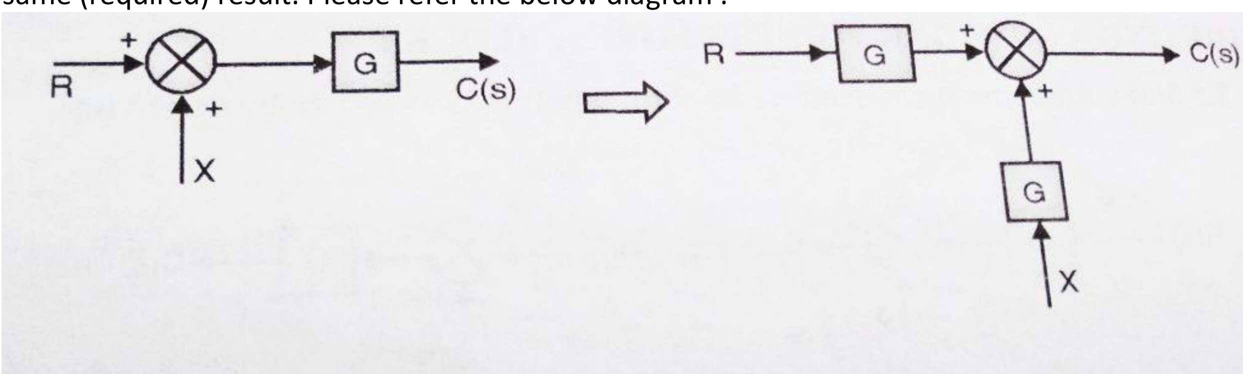


C(s)= GR(s)+ X
Aftershiftingthesummingpoint, wewillget
C(s)= [R+(X/G)] G= GR+ Xwhichissameasoutputinthefirst case.
Hencetoshiftasummingpointbeforeablock,weneedotoaddanotherblockoftransferfunction ‘1/G’ beforethesummingpointasshownin figure

.

Rule6:ShiftingoftheSummingPointaftera block

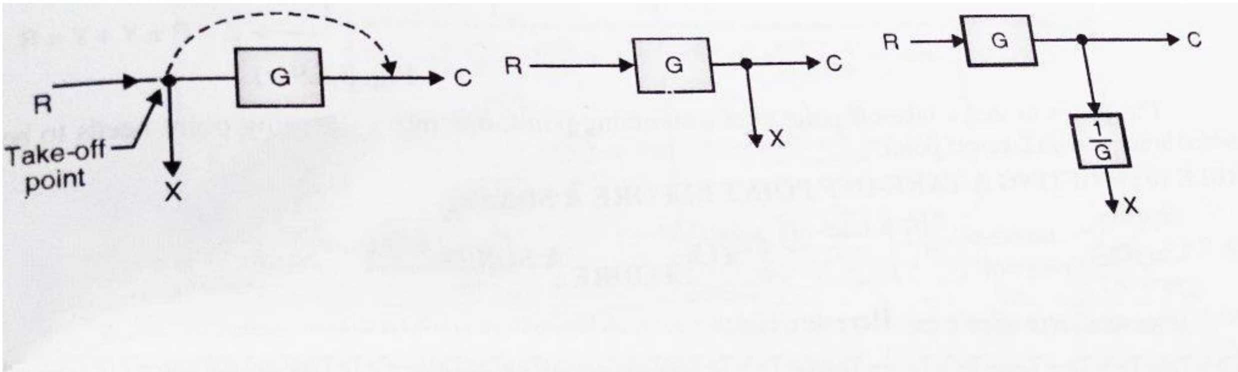
Whenwegenerallyshiftthesummingpointafteranyblock,werequiredtodothetransformationto attain the same (required) result. Please refer the below diagram .



C(s)= (R+ X)G
Aftershiftingthesummingpoint, wewillget
C(s)= (R+X)G= GR+XGwhichissameasoutputinthefirstcase.
Hence toshiftasummingpointbeforeablock,weneedtoadd anotherblockhavingthesametransfer functionatthe summing point as shown in fig

Rule6:ShiftingofTake-offpoint after ablock

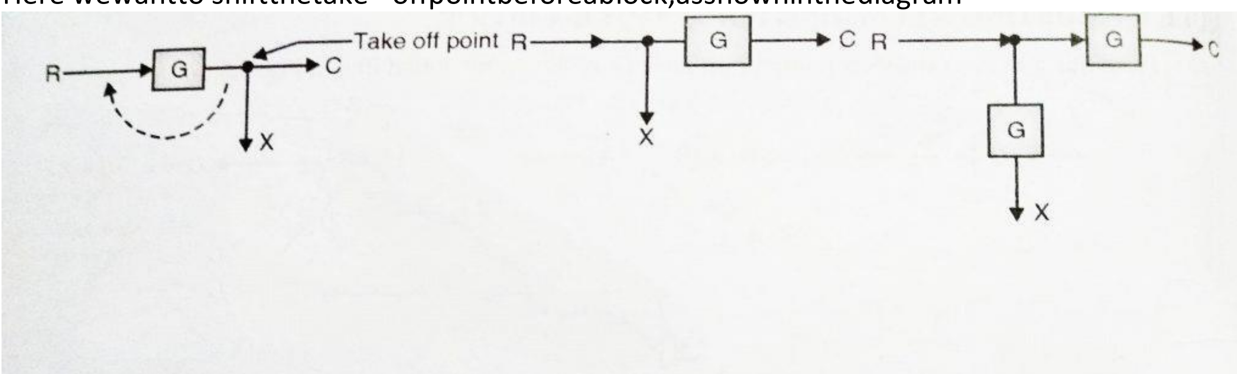
Herewewanttoshiftthetake–offpointafterablock, asshowninthe diagram



HerewehaveX=RandC=RG(initially)
In ordertoachievethis,weneed toaddablockof transferfunction ‘1/G’in serieswithsignaltakingoff from that point.

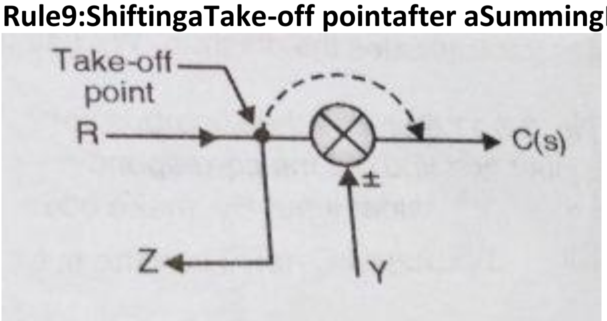
Rule8:ShiftingofTake-offpointbeforea block

Here wewantto shiftthetake– offpointbeforeablock,asshowninthediagram

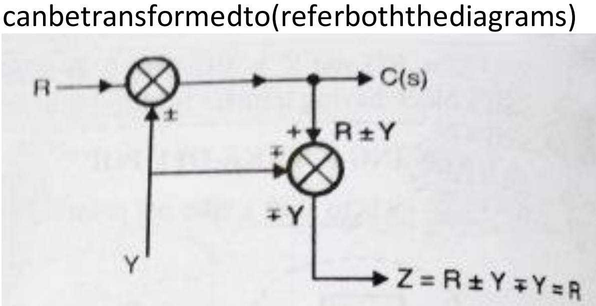


HerewehaveX=RandC=RG(initially)
In ordertoachievethis,weneed toaddablockof transferfunction ‘G’inserieswithXsignaltakingoff from that point.

Rule9:ShiftingaTake-off pointafter aSummingPoint



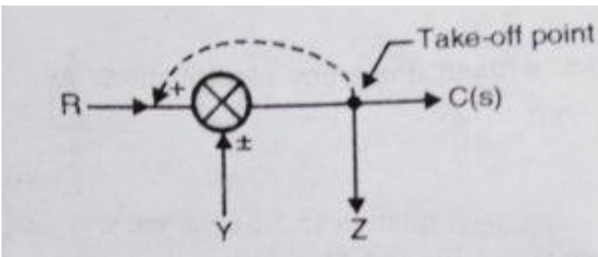
canbetransformedto(referboththediagrams)



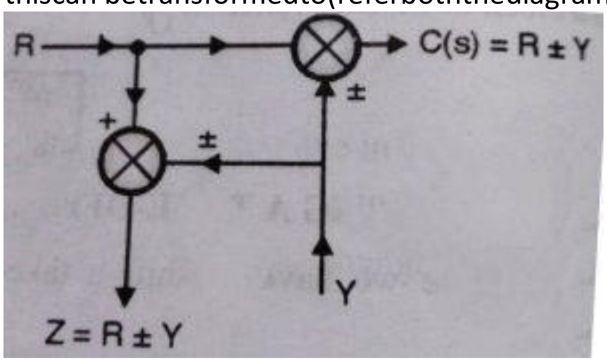
Beforeshiftingtake-offpoint,initially,wehave:
C(s)=R±Y
and Z=R ±Y(initially)
Hence ifwewanttoshiftatake-offpointafter asummingpoint,onemoresummingpoint needstobe added in series with take-off point.

Rule10:Shiftingatake-offpointbeforeasummingpoint

Supposeifwewanttoshifttake-offpointbeforeasummingpoint,theninitiallywehave C(s)=R±Y
and Z=R ±Y(initially)



thiscan betransformedto(referboththediagrams)



Inorderto satisfythiscondition,weneedtoadda summingpoint inserieswiththetake-off point.

4.4:Procedureforreductionofblockdiagram

Step-1-reducethecasecadeblocks
Step-2-reduce the parallel blocks
Step-3-Reducetheinternalfeedbackloop
Step-4-shifttakeoffpointtowardsrightandsummingpointtowardsleft
Step-5-repeat step 1 and step 4 untill the simple form is obtained
Step-6-findtransferfunctionoftheoverallsystemusingtheformula c(s)/r(s)

Procedureformultipleinput

Step-1-Herereduceallbutoneinputiszero.Findresultantoutput
Step-2- Reduce step 1 untill all input is covered
Step-3-Findtheresultantoutputbysuperposition

SimpleProblemforequivalenttransferfunction

Procedure for multiple in...

Step 1: Here reduce all but one input...

Step 2: Repeat step 1 until all inputs are cover...

Step 3: Find the resultant output by superposition.

Example 6.4 Determine the ratio $C(s)/R(s)$ of the block diagram shown in Fig. E5.4.

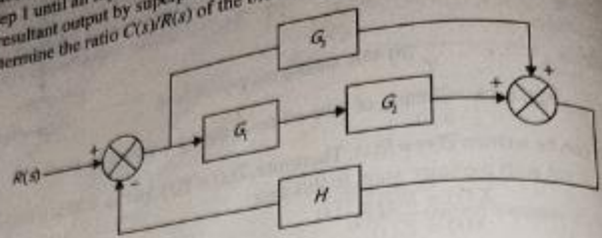


Fig. E6.4

Solution

G_1 and G_2 are connected in cascade and their equivalent is connected in parallel with G_3.

$$G = G_1G_2 + G_3$$

$$\therefore \quad \frac{C(s)}{R(s)} = \frac{G(s)}{1 - G(s)H(s)} = \frac{G_1G_2 + G_3}{1 - (G_1G_2 + G_3)H}$$

Example 6.5 Find $C(s)/R(s)$ of the block diagram shown in Fig. E6.5.

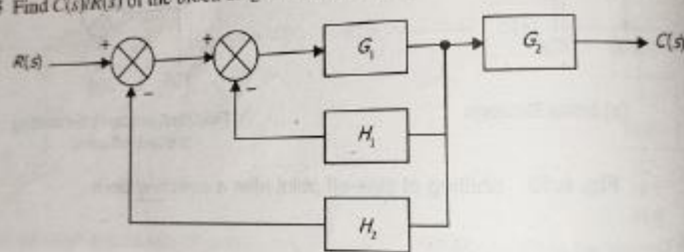


Fig. E6.5

Solution

Figure E6.5 is redrawn and shown in Fig. E6.5(a).

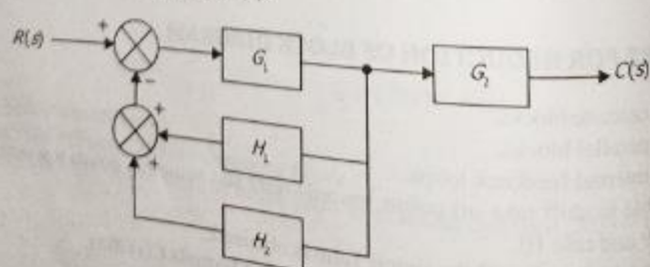


Fig. E6.5(a)

Since the two feedback loops are parallel, Fig. E6.5(a) becomes

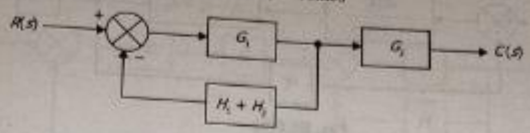


Fig. E6.5(b)

Replacing the feedback loop of Fig. E6.5(b) by its equivalent block, Fig. E6.5(b) becomes

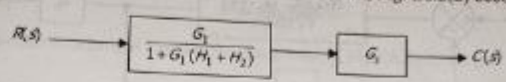


Fig. E6.5(c)

$$\therefore \qquad \frac{C(s)}{R(s)} = \frac{G_1G_2}{1+G_1(H_1+H_2)}$$

Example 6.6 Find the single block equivalent of Figure E6.6.

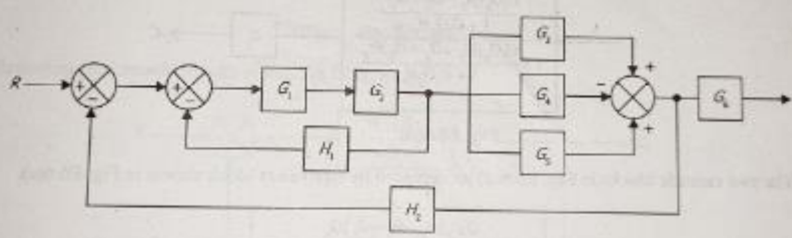


Fig. E6.6

Solution

At first the cascade and parallel blocks of Fig. E6.6 are reduced as shown in Fig. E6.6(a).

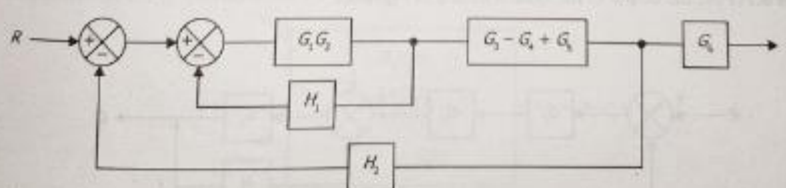
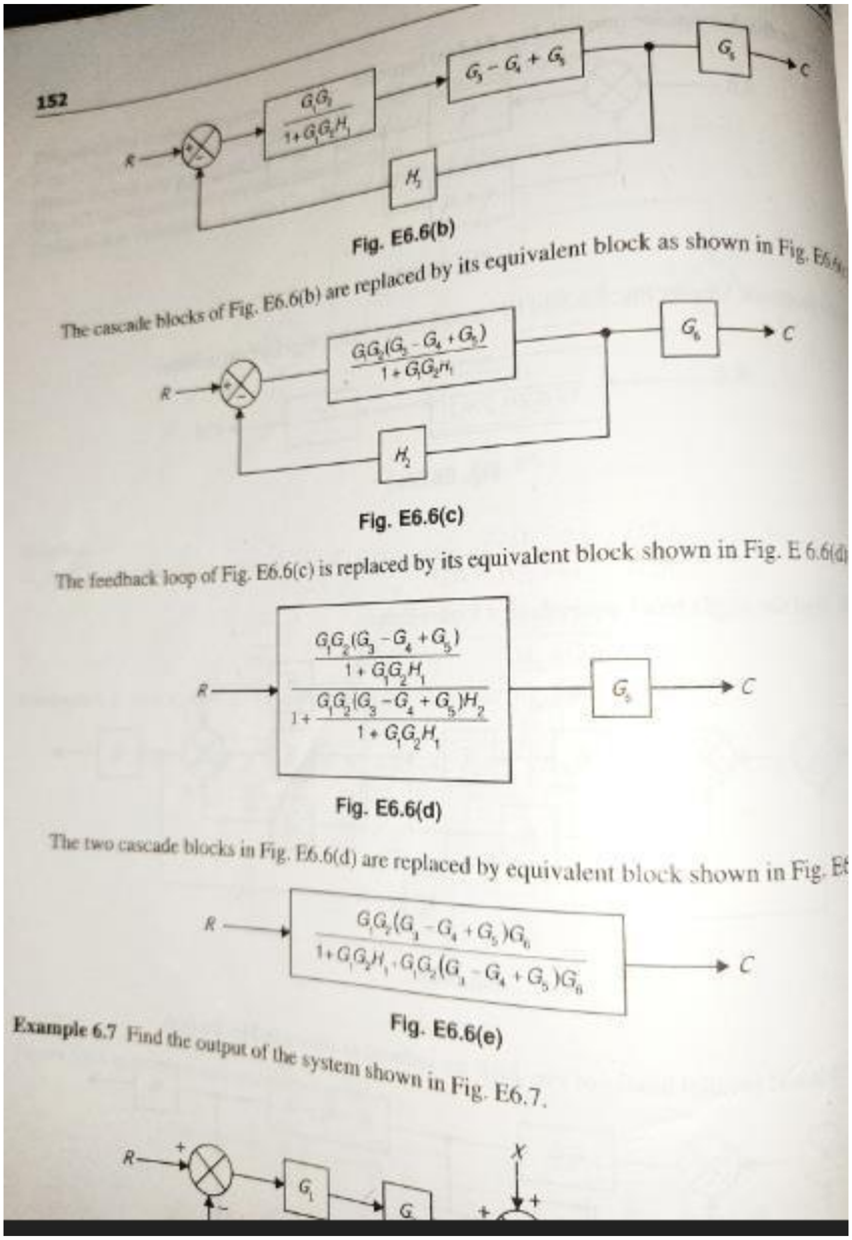


Fig. E6.6(a)

The first internal feedback loop of Fig. E6.6(a) is reduced by its equivalent block shown in Fig. E6.6(b).



152

Fig. E6.6(b)

The cascade blocks of Fig. E6.6(b) are replaced by its equivalent block as shown in Fig. E6.6

Fig. E6.6(c)

The feedback loop of Fig. E6.6(c) is replaced by its equivalent block shown in Fig. E 6.6(d)

Fig. E6.6(d)

The two cascade blocks in Fig. E6.6(d) are replaced by equivalent block shown in Fig. E6

Fig. E6.6(e)

Example 6.7 Find the output of the system shown in Fig. E6.7.

ControlSystems-SignalFlowGraphs

Signal flow graph is a graphical representation of algebraic equations. In this chapter, let us discuss the basic concepts related signal flow graph and also learn how to draw signal flow graphs.

BasicElementsofSignalFlowGraph

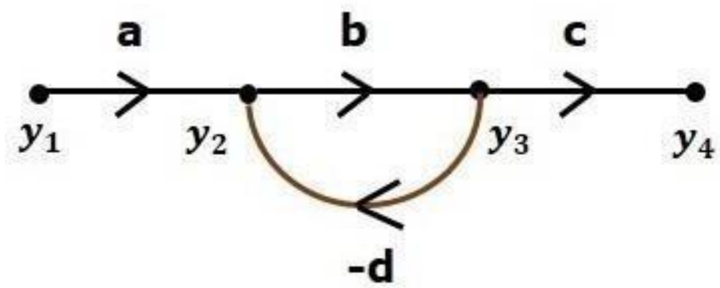
Nodesandbranchesarethebasicelementsofsignalflowgraph. Node

Nodeisapointwhichrepresentseitheravariableorasignal.Therearethreetypesofnodes—inputnode,outputnodeandmixednode.

- **InputNode**−Itisanode,whichhasonlyoutgoingbranches.
- **OutputNode**−Itisanode,whichhasonlyincomingbranches.
- **MixedNode**−Itisanode,whichhasbothincomingandoutgoingbranches.

Example

Letusconsiderthefollowingsignalflowgraphtoidentifythesenodes.



- The**nodes**presentinthissignalflowgraphare**y1,y2,y3**and**y4**.
- **y1**and**y4**arethe**inputnode**and**outputnode**respectively.
- **y2**and**y3**are**mixednodes**.

Branch

Branch is a line segment which joins two nodes. It has both **gain** and **direction**. For example, therearefourbranchesintheabovesignalflowgraph.Thesebrancheshave **gains**of**a,b,c**and **-d**.

ConstructionofSignalFlowGraph

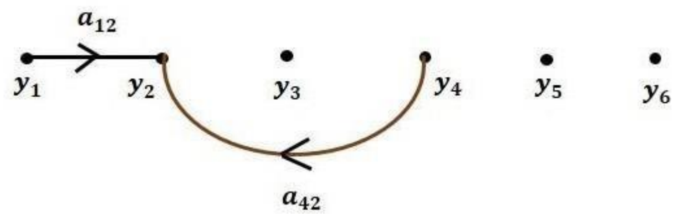
Letusconstructasignalflow graphbyconsideringthefollowingalgebraicequations−

$y2=a12y1+a42$
$y4$
$y3=a23y2+a53$
$y5$ $y4=a34y3$
$y5=a45y4+a35$
$y3$ $y6=a56y5$

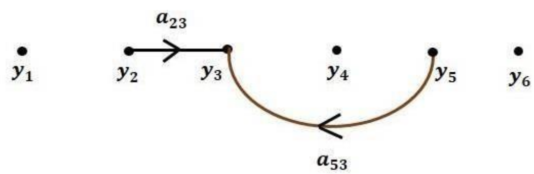
Therewillbesix**nodes**(y1,y2,y3,y4,y5andy6)andeight **branches**inthissignalflowgraph. The gains of the branches are a12, a23, a34, a45, a56, a42, a53 and a35.

Togettheoverallsignalflowgraph,drawthesignal flowgraphforeachequation,then combine all these signal flow graphs and then follow the steps given below −

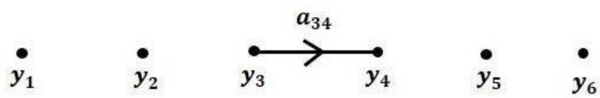
Step1−Signalflowgraphfor $y2=a13y1+a42y4$isshowninthefollowingfigure.



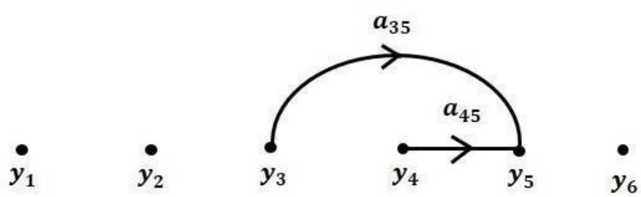
Step2−Signalflowgraphfor $y3=a23y2+a53y5$isshowninthefollowingfigure.



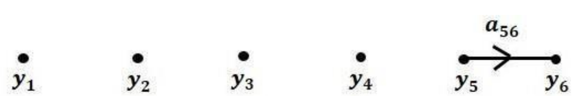
Step3−Signalflowgraphfor$y4=a34y3$isshowninthefollowing figure.



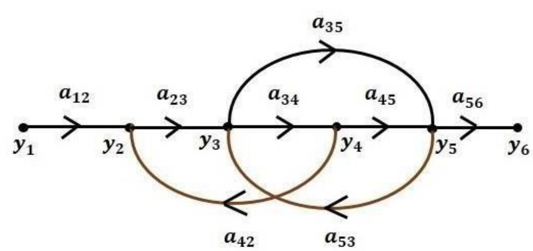
Step4−Signalflowgraphfor$y5=a45y4+a35y3$isshowninthefollowing figure.



Step5−Signalflowgraphfor$y6=a56y5$isshowninthefollowing figure.



Step6−Signalflow graphofoverallsystemisshowninthefollowingfigure.



ConstructionofSignalflow graphfromBlock Diagram

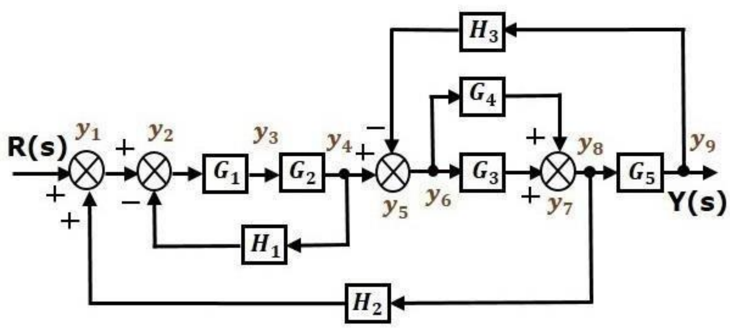
Followthesestepsforconvertingablockdiagramintoitsequivalentsignalflowgraph.

- Representallthesignals,variables,summingpointsandtake-offpointsofblock diagram as **nodes** in signal flow graph.
- Representtheblocksofblockdiagramas**branches**insignalflow graph.
-
- Representthetransferfunctionsinsidetheblocks ofblockdiagramas**gains**ofthebranchesin signal flow graph.

- Connectthe
-
- nodes as per the blockdiagram. Ifthere is connection between two nodes (butthere is no block in between), then represent the gain of the branch as one. **For example**, betweensummingpoints,betweensummingpointandtakeoffpoint,betweeninputand summing point, between take-off point andoutput.

Example

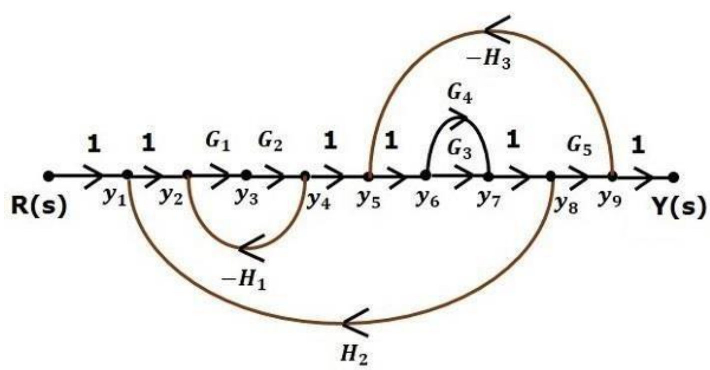
Letusconvertthefollowingblockdiagramintoitsequivalentsignalflowgraph.



Represent the input signal *R*(*s*) and output signal *C*(*s*) of block diagram as input node *R*(*s*) and output node *C*(*s*) of signal flow graph.

Just for reference, the remaining nodes (y1 to y9) are labelled in the block diagram. There are ninenodesotherthaninputandoutputnodes.Thatisfournodesforfoursummingpoints,four nodes for four take-off points and one node for the variable between blocks *G*1 and *G*2.

Thefollowingfigureshowstheequivalentsignalflowgraph.



With the help of Mason's gain formula (discussed in the next chapter), you can calculate the transferfunctionof thissignalflow graph.Thisistheadvantageof signalflow graphs.Here,we no need to simplify (reduce) the signal flow graphs for calculating the transfer function.

4.8Mason'sGainFormula

Let us now discuss the Mason’s Gain Formula. Suppose there are ‘N’ forward paths in a signal flow graph. The gain between the input and the output nodes of a signalflow graph is nothing but the **transfer function** of the system. It can be calculated by using Mason’s gain formula.

Mason’sgainformula is

$T=C(s)R(s)=\Sigma Ni=1 Pi\Delta i\Delta$

Where,

- **C(s)**istheoutputnode
- **R(s)**istheinputnode
- **T**isthetransferfunctionorgainbetween $R(s)$and$C(s)$
- **P**$_i$istheithforwardpathgain

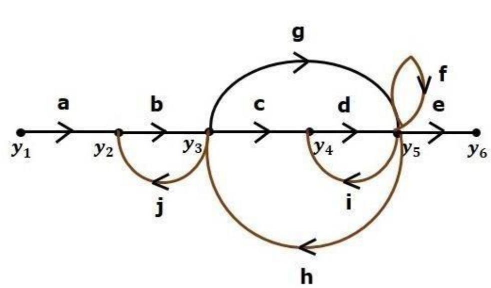
$\Delta=1-$(*sumofallindividualloopgains*)

+(*sumofgainproductsofallpossibletwonontouchingloops*)

−(*sumofgainproductsofallpossiblethreenontouchingloops*)+...

Δ_i*isobtainedfrom*Δ*byremovingtheloopswhicharetouchingthei*th*forwardpath*.

Considerthefollowingsignalflowgraphinordertounderstandthebasicterminologyinvolved here.



Path

Itisatraversalofbranchesfromonenodetoanyothernode inthedirectionofbrancharrows.It should not traverse any node more than once.

Examples−*y*2→*y*3→*y*4→*y*5and*y*5→*y*3→*y*2

ForwardPath

Thepaththatexistsfromtheinputnodetotheoutputnodeisknownas **forwardpath**. **Examples**

− *y*1→*y*2→*y*3→*y*4→*y*5→*y*6 and *y*1→*y*2→*y*3→*y*5→*y*6.

ForwardPathGain

Itisobtainedbycalculatingtheproductofallbranchgainsoftheforwardpath.

Examples−*abcde*istheforwardpathgainof *y*1→*y*2→*y*3→*y*4→*y*5→*y*6andabgeisthe forward path gain of *y*1→*y*2→*y*3→*y*5→*y*6.

Loop

Thepaththatstartsfromonenodeandendsatthesamenodeisknownas **loop**.Hence,itisa closed path.

Examples−*y*2→*y*3→*y*2and*y*3→*y*5→*y*3.

LoopGain

Itisobtainedbycalculatingtheproductofallbranchgainsofa loop.

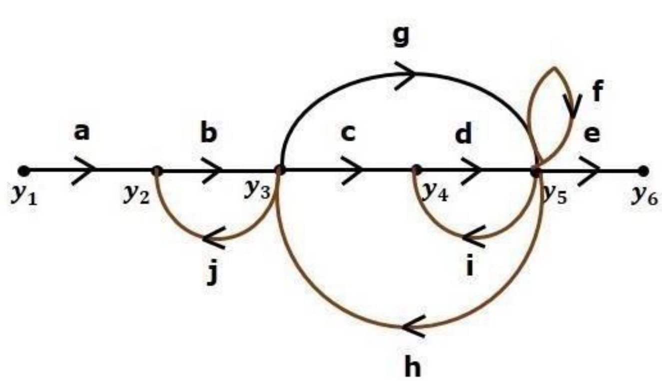
Examples−*bj*istheloopgainof*y*2→*y*3→*y*2and*gh*istheloopgainof*y*3→*y*5→*y*3.

Non-touchingLoops

Thesearetheloops,whichshouldnothaveanycommonnode.

Examples − The loops, *y*2→*y*3→*y*2 and *y*4→*y*5→*y*4 are non-touching.

CalculationofTransferFunctionusingMason'sGainFormula Letusconsider the same signal flow graph for finding transfer function.



- Numberofforwardpaths,N=2.
- Firstforwardpathis-$y_1 \rightarrow y_2 \rightarrow y_3 \rightarrow y_4 \rightarrow y_5 \rightarrow y_6$.
- Firstforwardpathgain,$p_1 = abcde$.
- Secondforwardpath is-$y_1 \rightarrow y_2 \rightarrow y_3 \rightarrow y_5 \rightarrow y_6$.
- Secondforwardpathgain,$p_2 = abge$.
- Numberofindividualloops,L=5.
- Loopsare-$y_2 \rightarrow y_3 \rightarrow y_2$,$y_3 \rightarrow y_5 \rightarrow y_3$,$y_3 \rightarrow y_4 \rightarrow y_5 \rightarrow y_3$,$y_4 \rightarrow y_5 \rightarrow y_4andy_5 \rightarrow y_5$.
- Loopgainsare-$l_1 = bj$,$l_2 = gh$,$l_3 = cdh$,$l_4 = di$and$l_5 = f$.
- Numberoftwonon-touchingloops=2.
- Firstnon-touchingloopspairis-$y_2 \rightarrow y_3 \rightarrow y_2$,$y_4 \rightarrow y_5 \rightarrow y_4$.
- Gainproductoffirstnon-touchingloopspair,$l_1 l_4 = bjdi$
- Secondnon-touchingloopspairis-$y_2 \rightarrow y_3 \rightarrow y_2$,$y_5 \rightarrow y_5$.
- Gainproductofsecondnon-touchingloopspairis-$l_1 l_5 = bjf$

Highernumberof(morethantwo)non-touchingloopsarenotpresentinthissignalflow graph.

We know,

$\Delta = 1 - (sumofallindividualloopgains)$

$+(sumofgainproductsofallpossibletwonontouchingloops)$

$-(sumofgainproductsofallpossiblethreenontouchingloops)+\ldots$

Substitute the values in the above equation,

$\Delta = 1 - (bj + gh + cdh + di + f) + (bjdi + bjf) - (0)$

$\Rightarrow \Delta = 1 - (bj + gh + cdh + di + f) + bjdi + bjf$

Thereisnoloopwhichisnon-touchingtothefirstforward path.

So,$\Delta_1 = 1$.

Similarly,$\Delta_2 = 1$.Since,noloopwhichisnon-touchingtothesecondforward path.

Substitute,N=2inMason'sgain formula

$T = \frac{C(s)}{R(s)} = \frac{\Sigma_{i=1}^{2} P_i \Delta_i}{\Delta}$

$T = \frac{C(s)}{R(s)} = \frac{P_1 \Delta_1 + P_2 \Delta_2}{\Delta}$

Substituteallthenecessaryvaluesintheaboveequation.

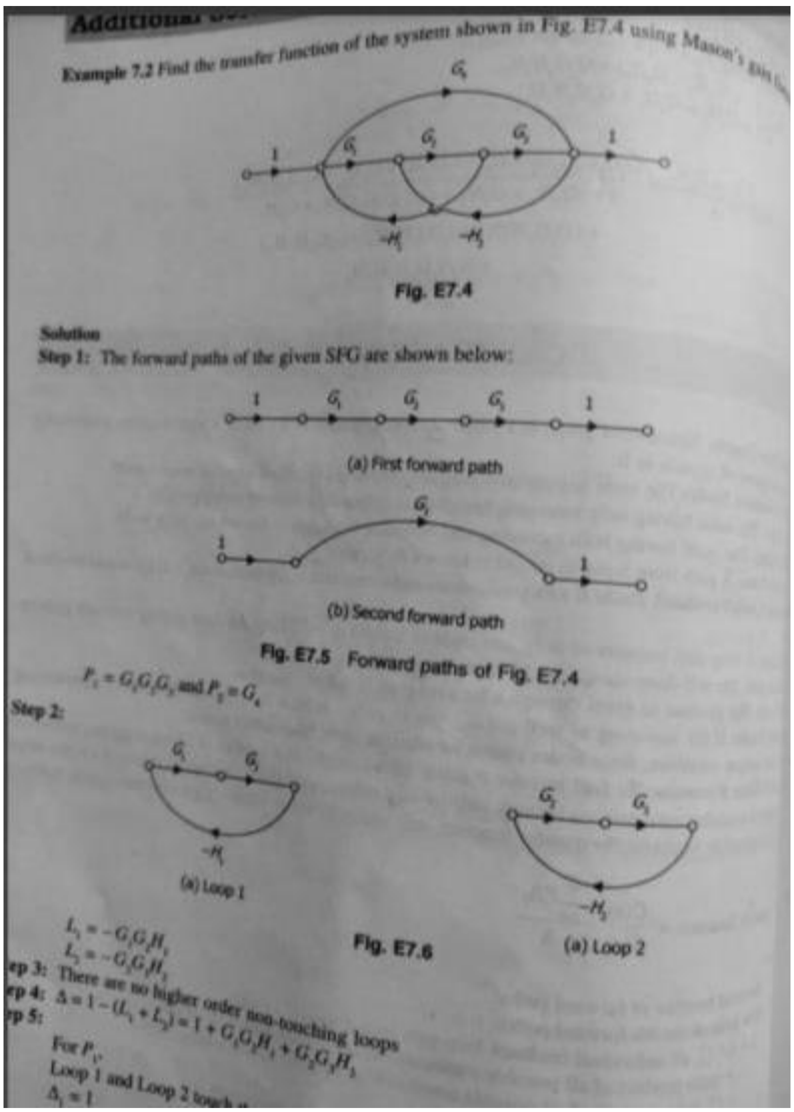
$T = \frac{C(s)}{R(s)} = \frac{(abcde)1 + (abge)1}{1 - (bj + gh + cdh + di + f) + bjdi + bjf}$

$\Rightarrow T = \frac{C(s)}{R(s)} = \frac{(abcde) + (abge)}{1 - (bj + gh + cdh + di + f) + bjdi + bjf}$

Therefore,thetransferfunctionis -

$T=C(s)R(s)=(abcde)+(abge)1-(bj+gh+cdh+di+f)+bjdi+bjf$

4.9.Simpleproblemsinsignalflowgraphfornetwork



Additional ...

Example 7.2 Find the transfer function of the system shown in Fig. E7.4 using Mason's gain ...

Fig. E7.4

Solution

Step 1: The forward paths of the given SFG are shown below:

(a) First forward path

(b) Second forward path

Fig. E7.5 Forward paths of Fig. E7.4

$P_1 = G_1G_2G_3$ and $P_2 = G_4$

Step 2:

(a) Loop 1

Fig. E7.6

(a) Loop 2

$L_1 = -G_1G_2H_1$

$L_2 = -G_2G_3H_2$

Step 3: There are no higher order non-touching loops

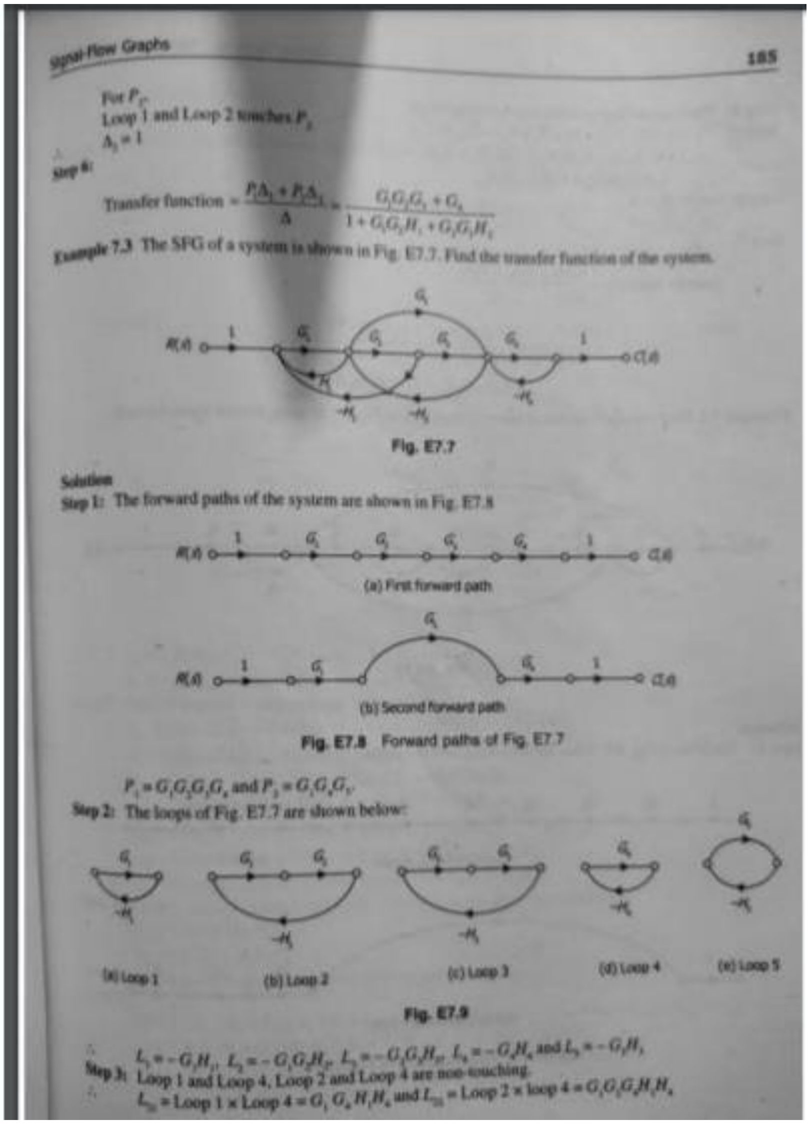
Step 4: $\Delta = 1 - (L_1 + L_2) = 1 + G_1G_2H_1 + G_2G_3H_2$

Step 5:

For P_1,

Loop 1 and Loop 2 touch ...

$\Delta_1 = 1$


Signal-Flow Graphs 185

For P_2,
Loop 1 and Loop 2 touches P_2
$\Delta_2 = 1$

Step 6:

Transfer function $= \frac{P_1\Delta_1 + P_2\Delta_2}{\Delta} = \frac{G_1G_2G_3 + G_4}{1 + G_1G_2H_1 + G_2G_3H_2}$

Example 7.3 The SFG of a system is shown in Fig. E7.7. Find the transfer function of the system.

Fig. E7.7

Solution

Step 1: The forward paths of the system are shown in Fig. E7.8

(a) First forward path

(b) Second forward path

Fig. E7.8 Forward paths of Fig. E7.7

$P_1 = G_1G_2G_3G_4$ and $P_2 = G_1G_5G_4$.

Step 2: The loops of Fig. E7.7 are shown below:

(a) Loop 1 (b) Loop 2 (c) Loop 3 (d) Loop 4 (e) Loop 5

Fig. E7.9

$L_1 = -G_1H_1$, $L_2 = -G_1G_2H_2$, $L_3 = -G_2G_3H_3$, $L_4 = -G_4H_4$ and $L_5 = -G_5H_5$

Step 3: Loop 1 and Loop 4, Loop 2 and Loop 4 are non-touching.
L_{14} = Loop 1 × Loop 4 $= G_1G_4H_1H_4$ and L_{24} = Loop 2 × loop 4 $= G_1G_2G_4H_2H_4$

Short questions

1whatdoyoumeanbyblock diagram

Ans A block diagram is a diagram of a system in which the principal parts or functions arerepresentedbyblocksconnectedbylinesthatshowtherelationshipsoftheblocks.[

2-whatdoumeanbysummingpointinfeedbackcontrol system

Ans-thesummingpointis representedwithacirclehavingcrossinsideit.Ithas twoor more inputand single output. It produce the algebraic sum of inputs. It also perform thesummationorsubtractionorcombinationofsummationandsubtractionofinputs based on the polarity of the inputs

3-whatissignalflowgraph

Agraphicalmethodofrepresentingthecontrolsystemusingthelinearalgebraic equations is known as the signal flow graph. It is abbreviated asSFG. This graph basically signifies how the signal flows in a system.

LONGQUESTIONS

1-statetherulesofblockdiagramreduction?

2-Writedownprocedureofreductionofblockdiagram?

3-writedownthestepsforsolvingsignalflow graph?

*4-writedownthestepsforfi**ndingtransferfunctionofasystemthroughmasngain formula?***

CHAPTER-5
TIMERESPONSEANALYSIS

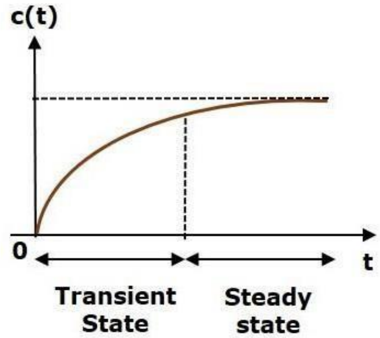
TIMERESPONSE ANALYSIS

5.1. WhatisTime Response?

Iftheoutputofcontrolsystemforaninputvarieswithrespecttotime,thenitiscalledthe **timeresponse**ofthecontrolsystem.Thetimeresponseconsistsoftwo parts.

- Transientresponse
- Steadystateresponse

Theresponseof controlsystemin timedomainisshowninthefollowingfigure.



Here,boththetransientandthesteadystatesareindicatedinthefigure.Theresponses corresponding to these states are known as transient and steady state responses.

Mathematically,wecanwritethetimeresponsec(t)as

$$c(t)=c_{tr}(t)+c_{ss}(t)c(t)=c_{tr}(t)+c_{ss}(t)$$

Where,

- $c_{tr}(t)$isthetransientresponse
- $c_{ss}(t)$isthesteadystateresponse

Transient Response

After applying input to the control system, output takes certain time to reach steady state. So, theoutputwill be intransient statetill it goestoa steady state.Therefore, theresponse ofthe control system during the transient state is known as **transient response**.

Thetransientresponsewillbezeroforlargevaluesof't'.Ideally,thisvalueof't'isinfinityand practically,it isfivetimesconstant.

Mathematically,wecan
write it as

$$\lim_{t\to\infty}c_{tr}(t)=0\lim_{t\to\infty}c_{tr}(t)=0$$

SteadystateResponse

Thepartofthetimeresponsethatremainsevenafterthetransientresponsehaszerovalue forlargevaluesof‘t’ is known as **steady state response**. This means, the transient response will be zero even during the steadystate.

5.2STANDARDTEST SIGNALS

Thestandardtestsignalsareimpulse,step,rampandparabolic.Thesesignalsareusedto know the performance of the control systems using time response of the output.

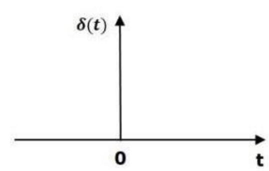
UnitImpulse Signal

Aunitimpulsesignal,δ(t)isdefinedas

δ(t)=0δ(t)=0fort≠0t≠0

and∫0+0−δ(t)dt=1∫0−0+δ(t)dt=1

Thefollowingfigureshowsunitimpulse signal.



So, theunit impulsesignal exists only at ‘t’ isequal tozero.The areaof this signalunder small intervaloftimearound ‘t’isequaltozeroisone.Thevalueofunitimpulsesignaliszeroforall other values of ‘t’.

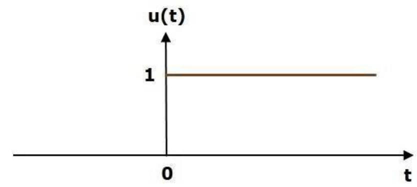
UnitStepSignal

Aunitstepsignal,u(t)isdefinedas

u(t)=1;t≥0u(t)=1;t≥0

=0;t<0=0;t<0

Followingfigureshowsunitstepsignal.



So,theunitstepsignalexists forallpositivevaluesof‘t’includingzero.Anditsvalueisone during this interval. Thevalue ofthe unit step signal is zero forallnegativevalues of ‘t’.

UnitRampSignal

Aunitrampsignal,r(t)isdefinedas

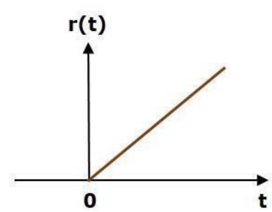
r(t)=t;t≥0r(t)=t;t≥0

=0;t<0=0;t<0

Wecanwriteunitrampsignal,r(t)r(t)intermsofunitstepsignal,u(t)u(t)as

r(t)=tu(t)r(t)=tu(t)

Followingfigureshowsunitrampsignal.



So,theunitrampsignalexistsforallpositivevaluesof‘t’includingzero.Andits value increaseslinearlywithrespectto‘t’duringthisinterval.Thevalueofunitrampsignaliszero for all negative values of ‘t’.

UnitParabolic Signal

Aunitparabolicsignal,p(t)isdefinedas,

p(t)=t22;t≥0p(t)=t22;t≥0

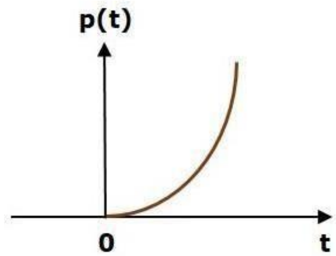
=0;t<0=0;t<0

Wecanwriteunitparabolicsignal,p(t)p(t)intermsoftheunitstepsignal, u(t)u(t)as,

p(t)=t22u(t)p(t)=t22u(t)

Thefollowingfigureshowstheunitparabolicsignal.

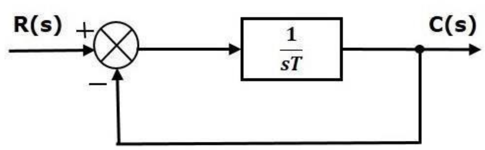
Thefollowingfigureshowstheunitparabolic signal.



So,theunitparabolicsignalexistsforallthepositivevaluesof **‘t’**includingzero.Anditsvalue increases non-linearly with respect to ‘t’ during this interval. The value of the unit parabolic signal is zero for all the negative values of ‘t’.

5.3ResponseoftheFirstOrderSystem

Inthis chapter,let us discuss the time response ofthe first order system. Consider the followingblockdiagramoftheclosedloopcontrolsystem.Here,anopenlooptransfer function, 1sT1sT is connected with a unity negative feedback.



Weknowthatthetransferfunctionoftheclosedloopcontrolsystemhasunitynegativefeedbackas,

C(s)R(s)=G(s)1+G(s)C(s)R(s)=G(s)1+G(s)

Substitute,G(s)=1sTG(s)=1sTintheaboveequation.

C(s)R(s)=1sT1+1sT=1sT+1C(s)R(s)=1sT1+1sT=1sT+1

Thepowerofsisoneinthedenominatorterm.Hence,the abovetransferfunctionisofthe first order and the system is said to be the **first ordersystem**.

Wecanre-writetheabove equation as

C(s)=(1sT+1)R(s)C(s)=(1sT+1)R(s)

Where,

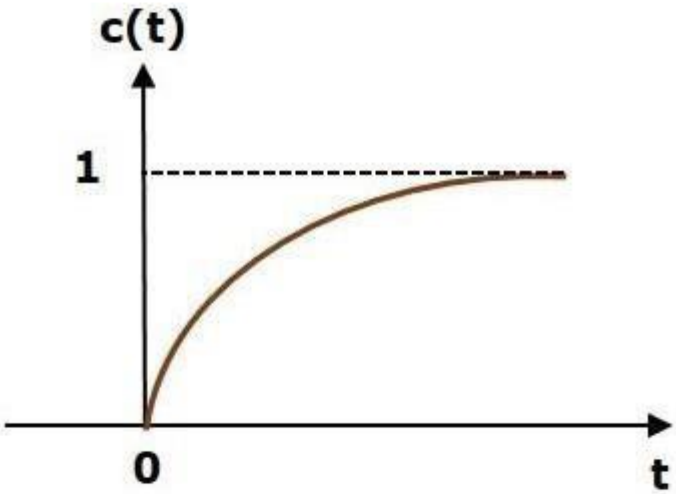
- **C(s)**istheLaplacetransformoftheoutputsignalc(t),
- **R(s)**istheLaplacetransformoftheinputsignalr(t),and
- **T** isthetimeconstant.

Followthesestepstogettheresponse(output)ofthefirstordersysteminthetime domain.

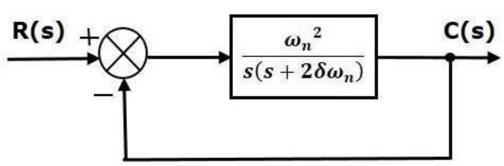
- TaketheLaplacetransformoftheinputsignalr(t)r(t).
- Considertheequation,C(s)=(1sT+1)R(s)C(s)=(1sT+1)R(s)
- SubstituteR(s)R(s)valueintheaboveequation.
- Dopartialfractions ofC(s)C(s)ifrequired.
- ApplyinverseLaplacetransformtoC(s)C(s).
- In theprevious chapter,wehave seen the standardtest signalslike impulse, step, ramp andparabolic.Letusnowfindouttheresponsesofthefirstordersystemforeachinput, one by one. The name of the response is given as per the name of the input signal. For example,the responseof the systemforan impulse inputiscalled asimpulse response.
- StepResponseofFirstOrderSystem
- Considerthe **unitstepsignal**asaninputtofirstordersystem.
- So,r(t)=u(t)r(t)=u(t)
- ApplyLaplacetransformonboththesides.
  - R(s)=1sR(s)=1s
- Considertheequation,C(s)=(1sT+1)R(s)C(s)=(1sT+1)R(s)
- Substitute,R(s)=1sR(s)=1sintheabove equation.
- C(s)=(1sT+1)(1s)=1s(sT+1)C(s)=(1sT+1)(1s)=1s(sT+1)
- Do partial fractionsof C(s).
  - C(s)=1s(sT+1)=As+BsT+1C(s)=1s(sT+1)=As+BsT+1
  - ⇒1s(sT+1)=A(sT+1)+Bss(sT+1)⇒1s(sT+1)=A(sT+1)+Bss(sT+1)
- Onboththesides,thedenominatortermisthe same.So,theywillgetcancelledby each other. Hence, equate the numerator terms.
  - 1=A(sT+1)+Bs1=A(sT+1)+Bs

- By equating the constant terms on both the sides,youwillgetA=1.Substitute,A=1and equatethecoefficientofthe **s**termsonboth the sides.
    - 0=T+B⇒B=−T0=T+B⇒B=−T
- Substitute,A=1andB=−Tinpartialfractionexpansionof C(s)C(s).
    - C(s)=1s−TsT+1=1s−TT(s+1T)C(s)=1s−TsT+1=1s−TT(s+1T)
    - ⇒C(s)=1s−1s+1T⇒C(s)=1s−1s+1T
- ApplyinverseLaplacetransformonboththesides.
    - c(t)=(1−e−(tT))u(t)c(t)=(1−e−(tT))u(t)
- The **unit step response**, c(t) has both the transient and the steady state terms. The transienttermintheunitstepresponseis-
    - ctr(t)=−e−(tT)u(t)ctr(t)=−e−(tT)u(t)

- Thesteady stateterm in theunit stepresponse is–
- Thetransientterm intheunitstep responseis-
    - ctr(t)=−e−(tT)u(t)ctr(t)=−e−(tT)u(t)
-
- Thefollowing figureshows theunit stepresponse.
-



-
-
-
- Thevalueofthe**unitstepresponse,c(t)** iszeroatt=0andforallnegativevalues of t.Itisgraduallyincreasingfromzerovalueandfinallyreachestooneinsteadystate. So, the steady state value depends on the magnitude of the input.
- *ResponseofSecondOrderSystem*
- Inthischapter,let usdiscussthetimeresponseofsecond ordersystem.Considerthe



followingblockdiagramofclosedlooptcontrolsystem.Here,anopenlooptransfer function, ω2ns(s+2δωn)ωn2s(s+2δωn) is connected with a unity negative feedback.

-
-
- Weknowthatthetransferfunctionoftheclosedloopcontrolsystemhavingunitynegative feedback as

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1 + G(s)}$$

-

$$\frac{C(s)}{R(s)} = \frac{\left(\frac{\omega_n^2}{s(s+2\delta\omega_n)}\right)}{1 + \left(\frac{\omega_n^2}{s(s+2\delta\omega_n)}\right)} = \frac{\omega_n^2}{s^2 + 2\delta\omega_n s + \omega_n^2}$$

-
- Thepowerof's'istwointhedenominatorterm.Hence,theabovetransferfunctionisof the second order and the system is said to be the **second order system**.
- $s^2 + 2\delta\omega_n s + \omega_n^2 = 0$
- Thecharacteristicequation is

$$\Rightarrow s = -\delta\omega_n \pm \omega_n\sqrt{\delta^2 - 1}$$

Therootsofcharacteristicequationare -

- Thetwo rootsareimaginarywhen $\delta=0$.
- Thetwo rootsarerealand equalwhen $\delta=1$.
- Thetwo rootsarerealbutnotequalwhen $\delta>1$.
- Thetwo rootsarecomplex conjugatewhen $0 <\delta<1$.

5.4.1

Step Response of Second Order System

Consider the unit step signal as an input to the second order system.

Laplace transform of the unit step signal is,

$$R(s) = \frac{1}{s}$$

We know the transfer function of the second order closed loop control system is,

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\delta\omega_n s + \omega_n^2}$$

Case 1: δ = 0

Substitute, $\delta = 0$ in the transfer function.

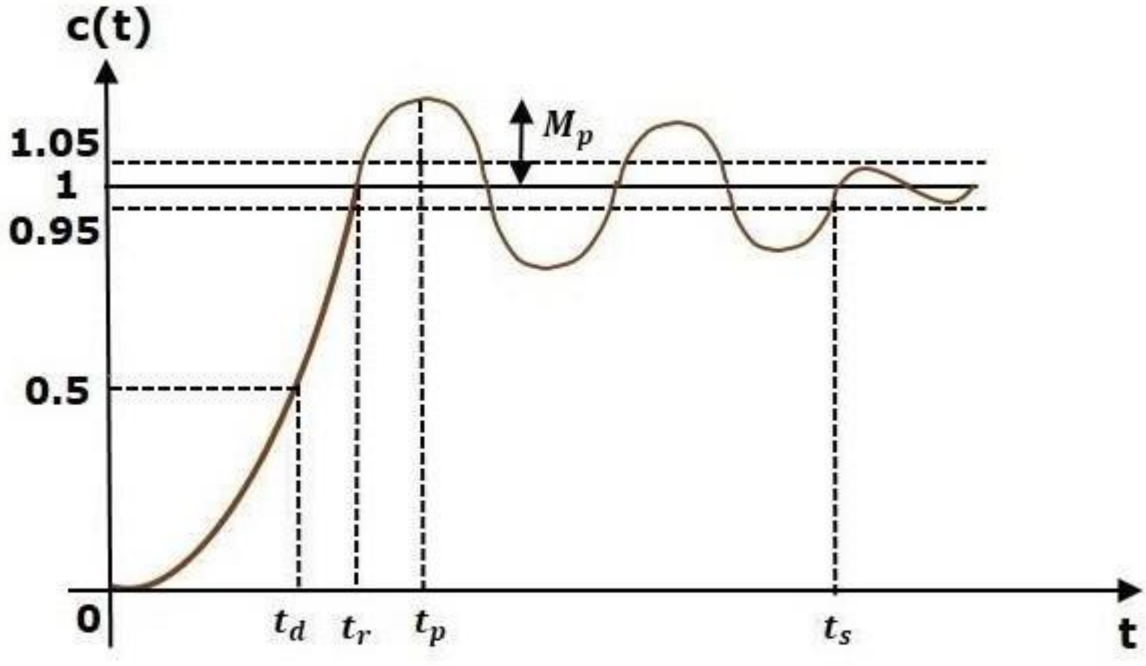
$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + \omega_n^2}$$

$$\Rightarrow C(s) = \left(\frac{\omega_n^2}{s^2 + \omega_n^2}\right) R(s)$$

Substitute, $R(s) = \frac{1}{s}$ in the above equation.

5.4.2TimeDomain Specifications

In this chapter, let us discuss the time domain specifications of the second order system. The stepresponseofthesecondordersystemfortheunderdampedcaseisshowninthe following figure.



Allthetimedomainspecificationsarerepresentedinthisfigure.Theresponseuptothesettling time is known as transient response and the response after the settling time is known as steady state response.

Delay Time

Itisthetimerequiredfortheresponsetoreach**halfofitsfinalvalue**fromthezero instant. It is denoted by tdtd. Consider the step response of the second order system for t ≥ 0, when ‘δ’ lies between zero and one.

$$c(t) = 1 - \left(\frac{e^{-\delta\omega_n t}}{\sqrt{1-\delta^2}}\right)\sin(\omega_d t + \theta)$$

Thefinal valueof thestep responseis one.

Therefore,att=t_dt=td,thevalueofthe stepresponsewillbe0.5.Substitute,thesevaluesinthe above

$$c(t_d) = 0.5 = 1 - \left(\frac{e^{-\delta\omega_n t_d}}{\sqrt{1-\delta^2}}\right)\sin(\omega_d t_d + \theta)$$

$$\Rightarrow \left(\frac{e^{-\delta\omega_n t_d}}{\sqrt{1-\delta^2}}\right)\sin(\omega_d t_d + \theta) = 0.5$$

equation.

Byusinglinearapproximation,youwillgetthe**delaytimet$_d$**as

$$t_d = \frac{1 + 0.7\delta}{\omega_n}$$

RiseTime

It is the time required for the response to rise from **0% to 100% of its final value**. This is applicable for the **under-damped systems**.Fortheover-dampedsystems,considerthedurationfrom10%to90%ofthefinalvalue. Risetimeisdenotedby **t_r**.

$$c(t) = 1 - \left(\frac{e^{-\delta\omega_n t}}{\sqrt{1-\delta^2}}\right)\sin(\omega_d t + \theta)$$

$$c(t_2) = 1 = 1 - \left(\frac{e^{-\delta\omega_n t_2}}{\sqrt{1-\delta^2}}\right)\sin(\omega_d t_2 + \theta)$$

$$\Rightarrow \left(\frac{e^{-\delta\omega_n t_2}}{\sqrt{1-\delta^2}}\right)\sin(\omega_d t_2 + \theta) = 0$$

$$\Rightarrow \sin(\omega_d t_2 + \theta) = 0$$

$$\Rightarrow \omega_d t_2 + \theta = \pi$$

$$\Rightarrow t_2 = \frac{\pi - \theta}{\omega_d}$$

Substitutet$_1$andt$_2$values inthefollowingequation of **rise time**,

$$t_r = t_2 - t_1$$

$$\therefore\ t_r = \frac{\pi - \theta}{\omega_d}$$

Fromaboveequation,wecanconcludethattherisetimet$_r$trandthedampedfrequencyω$_d$ωdare inversely proportionalto each other.

PeakTime

Itisthetimerequiredfortheresponsetoreachthe **peakvalue**forthefirsttime.Itisdenotedby t$_p$tp. At t=t$_p$t=tp, the first derivate of the response is zero.

Weknow thestep responseof secondorder system forunder-damped caseis

PeakOvershoot

Peakovershoot$M_p$isdefinedasthedeviationoftheresponseatpeaktimefromthefinal value ofresponse. Itis also calledthe **maximum overshoot**.

Mathematically,wecan write it as

$$M_p=c(t_p)-c(\infty)$$

$$M_p=c(t_p)-c(\infty)$$

Where,

$c(t_p)$is the peakvalue oftheresponse.

$c(\infty)$isthefinal(steady state)valueofthe response.

$$c(t_p) = 1 - \left(\frac{e^{-\delta \omega_n t_p}}{\sqrt{1-\delta^2}} \right) \sin(\omega_d t_p + \theta)$$

Substitute, $t_p = \frac{\pi}{\omega_d}$ in the right hand side of the above equation.

$$c(t_P) = 1 - \left(\frac{e^{-\delta \omega_n \left(\frac{\pi}{\omega_d}\right)}}{\sqrt{1-\delta^2}} \right) \sin\left(\omega_d \left(\frac{\pi}{\omega_d} \right) + \theta \right)$$

$$\Rightarrow c(t_p) = 1 - \left(\frac{e^{-\left(\frac{\delta\pi}{\sqrt{1-\delta^2}}\right)}}{\sqrt{1-\delta^2}} \right) (-\sin(\theta))$$

We know that

$$\sin(\theta) = \sqrt{1-\delta^2}$$

So, we will get $c(t_p)$ as

$$c(t_p) = 1 + e^{-\left(\frac{\delta\pi}{\sqrt{1-\delta^2}}\right)}$$

Settlingtime

It is the time required for the response to reach the steady state and stay within the specified tolerance bands around the final value. In general, the tolerance bands are 2% and 5%. The settling time is denoted by tsts.

Thesettlingtimefor5%tolerance bandis-

$$t_s = \frac{3}{\delta \omega_n} = 3\tau$$

The settling time for 2% tolerance band is -

$$t_s = \frac{4}{\delta \omega_n} = 4\tau$$

Where, τ is the time constant and is equal to $\frac{1}{\delta \omega_n}$.

Time domain specification	Formula	Substitution of values in Formula	Final value
Delay time	$t_d=\frac{1+0.7\delta}{\omega_n}$ td=1+0.7δωn	$t_d=\frac{1+0.7(0.5)}{2}$ td=1+0.7(0.5)2	t_d td=0.675 sec
Rise time	$t_r=\frac{\pi-\theta}{\omega_d}$ tr=π−θωd	$t_r=\frac{\pi-(\frac{\pi}{3})}{1.732}$ tr=π−(π3)1.732	t_r tr=1.207 sec
Peak time	$t_p=\frac{\pi}{\omega_d}$ tp=πωd	$t_p=\frac{\pi}{1.732}$ tp=π1.732	t_p tp=1.813 sec
% Peak overshoot	$\%M_p=\left(e^{-\left(\frac{\delta\pi}{\sqrt{1-\delta^2}}\right)}\right)\times 100\%$ %Mp=(e−(δπ1−δ2))×100%	$\%M_p=\left(e^{-\left(\frac{0.5\pi}{\sqrt{1-(0.5)^2}}\right)}\right)\times 100\%$ %Mp=(e−(0.5π1−(0.5)2))×100%	$\%M_p$ %Mp =16.32%
Settling time for 2% tolerance band	$t_s=\frac{4}{\delta\omega_n}$ ts=4δωn	$t_S=\frac{4}{(0.5)(2)}$ tS=4(0.5)(2)	t_s ts=4sec

5.4.3 Control Systems - Steady State Errors

The deviation of the output of control system from desired response during steady state is known as **steady state error**. It is represented as e_{ss} ess. We can find steady state error using the final value theorem as follows.

$$e_{ss}=\lim_{t\to\infty}e(t)=\lim_{s\to 0}E(s)$$ ess=limt→∞e(t)=lims→0E(s)

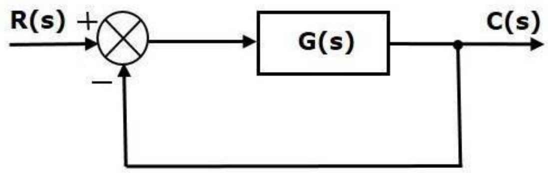
Where,

E(s) is the Laplace transform of the error signal, $e(t)$ e(t)

Let us discuss how to find steady state errors for unity feedback and non-unity feedback control systems one by one.

SteadyStateErrorsforUnityFeedbackSystems

Considerthefollowingblockdiagramofclosedloopcontrolsystem,whichishavingunitynegative feedback.



$$\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)}$$

$$\Rightarrow C(s) = \frac{R(s)G(s)}{1+G(s)}$$

The output of the summing point is -

$$E(s) = R(s) - C(s)$$

Substitute $C(s)$ value in the above equation.

$$E(s) = R(s) - \frac{R(s)G(s)}{1+G(s)}$$

$$\Rightarrow E(s) = \frac{R(s) + R(s)G(s) - R(s)G(s)}{1+G(s)}$$

$$\Rightarrow E(s) = \frac{R(s)}{1+G(s)}$$

Substitute $E(s)$ value in the steady state error formula

$$e_{ss} = \lim_{s \to 0} \frac{sR(s)}{1+G(s)}$$

Input signal	Steadystateerroressess	Errorconstant
unit step signal	11+kp11+kp	Kp=lims→0G(s)Kp=lims→0G(s)
unitrampsignal	1Kv1Kv	Kv=lims→0sG(s)Kv=lims→0sG(s)

unitparabolic signal	$\frac{1}{K_a}$1Ka	$K_a=\lim_{s\to 0}s^2G(s)$Ka=lims→0s2G(s)
Input signal	**Errorconstant**	**Steadystateerror**
$r_1(t)=5u(t)$r1(t)=5u(t)	$K_p=\lim_{s\to 0}G(s)=\infty$Kp=lims→0G(s)=∞	$e_{ss1}=\frac{5}{1+k_p}=0$ess1=51+kp=0
$r_2(t)=2tu(t)$r2(t)=2tu(t)	$K_v=\lim_{s\to 0}sG(s)=\infty$Kv=lims→0sG(s)=∞	$e_{ss2}=\frac{2}{K_v}=0$ess2=2Kv=0
$r_3(t)=\frac{t^2}{2}u(t)$r3(t)=t22u(t)	$K_a=\lim_{s\to 0}s^2G(s)=1$Ka=lims→0s2G(s)=1	$e_{ss3}=\frac{1}{k_a}=1$ess3=1ka=1

Wewillgettheoverallsteadystateerror, by addingtheabovethreesteadystateerrors.

$e_{ss}=e_{ss1}+e_{ss2}+e_{ss3}$ess=ess1+ess2+ess3

$\Rightarrow e_{ss}=0+0+1=1$⇒ess=0+0+1=1

5.5Typesofcontrolsystem(type0,type1,type2system)

Stepinput toaType0system:

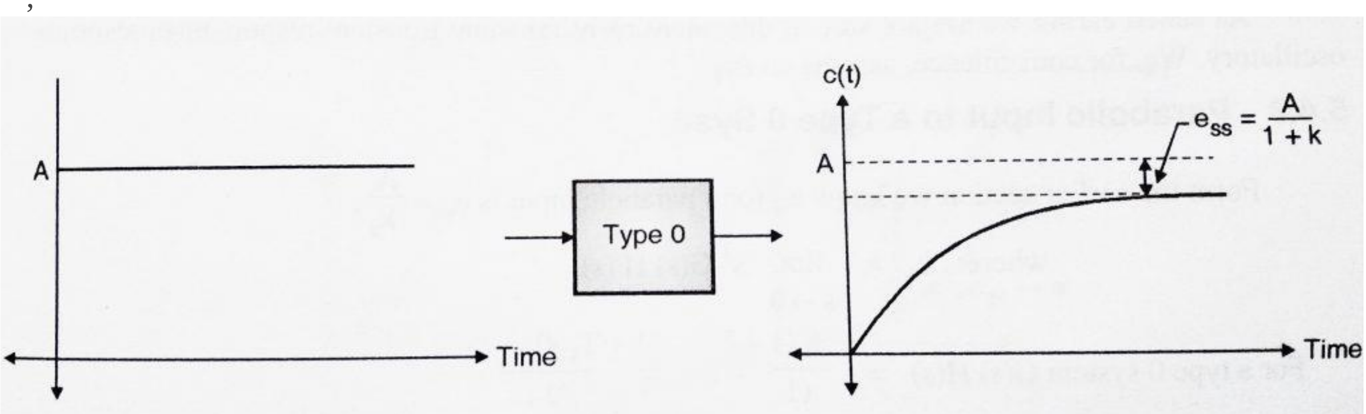
From my previous post , we already know the steady state error 'e_{ss}' for a step input is

$$e_{ss}=\frac{A}{1+k_p},$$

where $k_p=\lim_{s\to 0}G(s)H(s)=\lim_{s\to 0}\frac{k(1+T_{z1}s)(1+T_{z2}s)\ldots\ldots\ldots}{s^0(1+T_{p1}s)(1+T_{p2}s)\ldots\ldots\ldots}$

,

,



$$k_p = \frac{k(1)(1)....}{s^0(1)(1)....}$$

= k ,

$$e_{ss} = \frac{A}{1+k_p} = \frac{A}{1+k}$$,

Hence when a type 0 system is subjected to a step input, we get a constant steady state error.

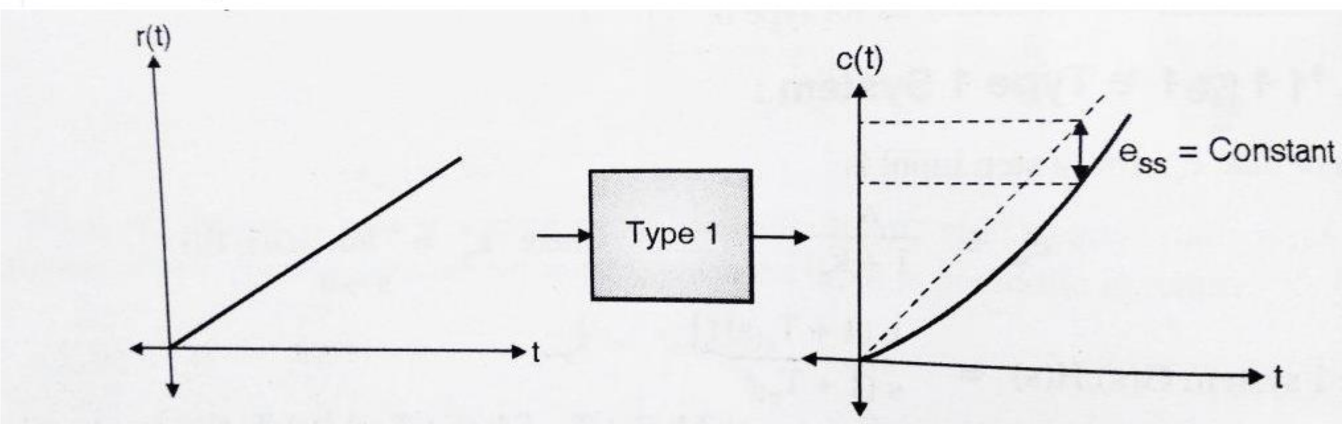
StepInputtoType1 System:

we already know 'e_{ss}' for a step input is

$$e_{ss} = \frac{A}{1+k_p} \quad \text{where} \quad k_p = \lim_{s \to 0} G(s)H(s) = \lim_{s \to 0} \frac{k(1+T_{z1}s)(1+T_{z2}s).........}{s^1(1+T_{p1}s)(1+T_{p2}s).........}$$,

$$k_p = \frac{k(1)(1)....}{s^1(1)(1)....}$$ = ∞ (infinity) ,

$$e_{ss} = \frac{A}{1+k_p} = \frac{A}{1+k}$$ = 0 ,



Hence when a type 1 system is subjected to a step input, we get steady state error i.e . .Hence we can conclude that type 1 systems are excellent for step inputs as steady state error is 0.

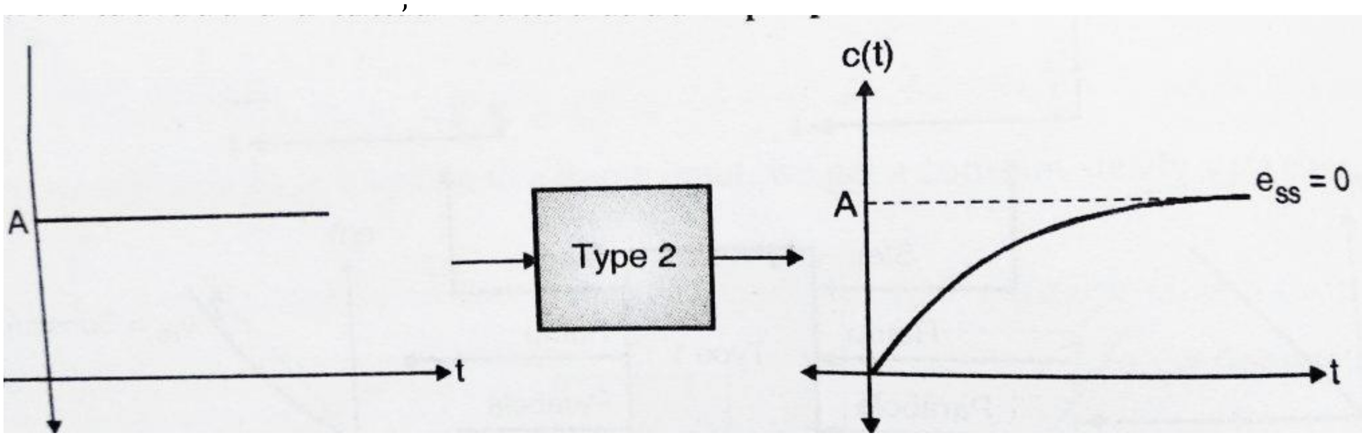
StepInputtoatype2System

we already know 'e_{ss}' for a step input is

$$e_{ss} = \frac{A}{1+k_p} \quad \text{where} \quad k_p = \lim_{s \to 0} G(s)H(s) = \lim_{s \to 0} \frac{k(1+T_{z1}s)(1+T_{z2}s)\ldots\ldots\ldots}{s^2(1+T_{p1}s)(1+T_{p2}s)\ldots\ldots\ldots} = 0 \quad ,$$

$$k_p = \frac{k(1)(1)\ldots}{s^2(1)(1)\ldots} = \infty \text{ (infinity) },$$

$$e_{ss} = \frac{A}{1+k_p} = \frac{A}{1+k} = 0 \quad ,$$



Hence when a type 2 system is subjected to a step input, we get steady state error i.e . =0.Hencewe can conclude that type 2 systems are excellent for step inputs as steady state error is 0.

RampinputtoaType 0system:

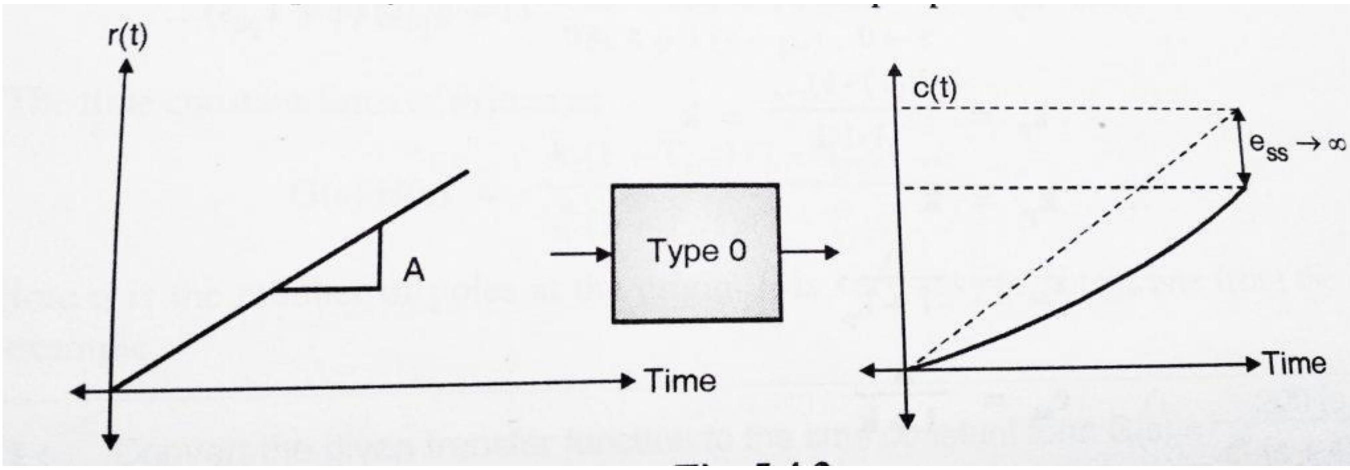
From my previous post , we already know the steady state error 'e_{ss}' for a ramp input is

$$e_{ss} = \frac{A}{k_v}$$

$$\text{where} \quad k_v = \lim_{s \to 0} sG(s)H(s) = \lim_{s \to 0} \frac{sk(1+T_{z1}s)(1+T_{z2}s)\ldots\ldots\ldots}{s^0(1+T_{p1}s)(1+T_{p2}s)\ldots\ldots\ldots} = 0,$$

$$k_v = \frac{sk(1)(1)\ldots}{s^0(1)(1)\ldots} = 0 \,,$$

$$e_{ss} = \frac{A}{k_v} = \frac{A}{k} = \infty \text{(infinity) },$$



Hencewhenwesubjectatype0systemtoarampinput,thesteadystateerrorincreasescontinuously.

RampinputtoaType 1System

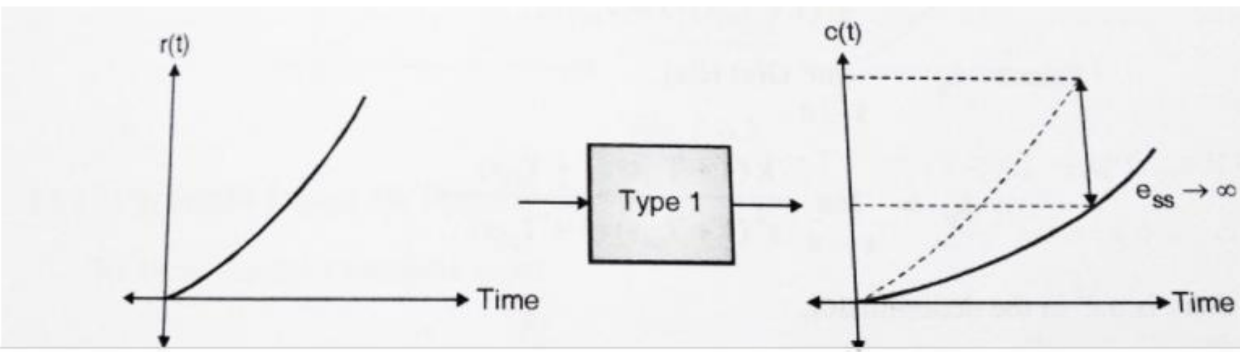
$e_{ss} = \frac{A}{k_v}$

where $k_v = \lim_{s \to 0} sG(s)H(s) = \lim_{s \to 0} \frac{sk(1+T_{z1}s)(1+T_{z2}s)\ldots\ldots\ldots}{s^1(1+T_{p1}s)(1+T_{p2}s)\ldots\ldots\ldots}$,

$k_v = \frac{sk(1)(1)\ldots}{s^1(1)(1)\ldots}$ = k ,

$e_{ss} = \frac{A}{k_v} = \frac{A}{k}$ = constant ,

Hence when we subject a type 1 system to a ramp input, the steady state error is constant.



3.2 Ramp input to a Type 2 System

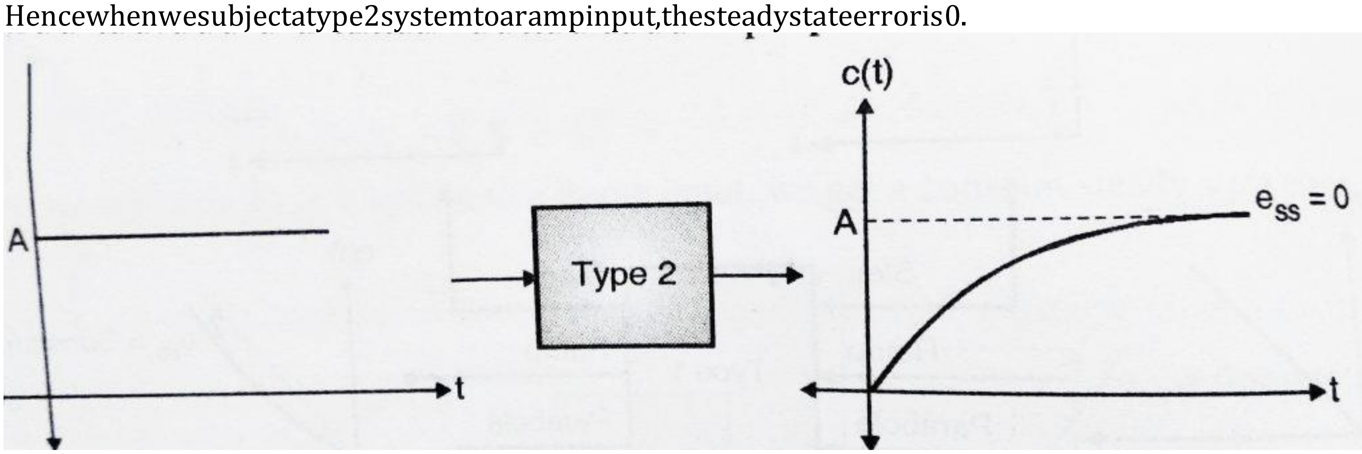
we already know 'e_{ss}' for a ramp input is

$e_{ss} = \frac{A}{k_v}$ where ,

$$k_v = \lim_{s \to 0} sG(s)H(s) = \lim_{s \to 0} \frac{sk(1+T_{z1}s)(1+T_{z2}s)\ldots\ldots\ldots}{s^2(1+T_{p1}s)(1+T_{p2}s)\ldots\ldots\ldots}$$

$k_v = \frac{sk(1)(1)\ldots}{s^2(1)(1)\ldots}$ = ∞ ,

$e_{ss} = \frac{A}{k_v} = \frac{A}{k}$ = 0 , Hencewhenwesubjectatype2systemtoarampinput,thesteadystateerroris0.



ParabolicinputtoaType0 system:

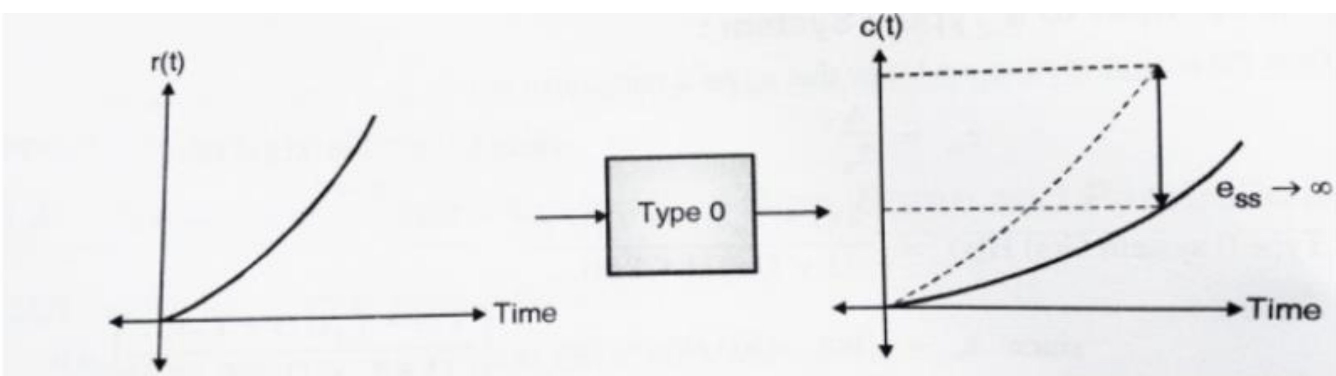
we already know 'e_{ss}' for a parabolic input is

$e_{ss} = \frac{A}{k_a}$ where ,

$$k_a = \lim_{s \to 0}(s^2 G(s)H(s) = \lim_{s \to 0} \frac{s^2 k(1+T_{z1}s)(1+T_{z2}s).........}{s^2(1+T_{p1}s)(1+T_{p2}s).........}$$

$k_a = \frac{s^2 k(1)(1)....}{s^2(1)(1)....}$ = k ,

$e_{ss} = \frac{A}{k_a} = \frac{A}{k}$ = const ,



ence when we subject a type 0 system to a parabolic input, the steady state error increases continuously.
ence type 0 system are not suitable when the input is parabolic in nature.

ow we shall shift our focus to type 1 systems:

'Type 1' system is given by

$'(s)H(s) = \lim_{s \to 0} \frac{k(1+T_{z1}s)(1+T_{z2}s).........}{s^1(1+T_{p1}s)(1+T_{p2}s).........}$ = 0 (one pole at origin)

ParabolicInputtoaType1System:

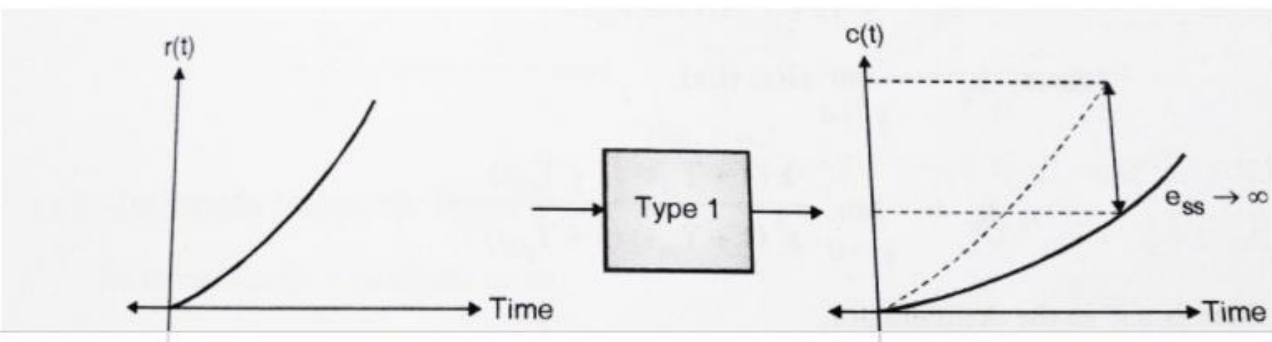
we already know 'e_{ss}' for a parabolic input is

$$e_{ss} = \frac{A}{k_a} \quad \text{where} \quad k_a = \lim_{s\to0} s^2 G(s)H(s) = \lim_{s\to0} \frac{s^2 k(1+T_{z1}s)(1+T_{z2}s)\ldots\ldots\ldots}{s^1(1+T_{p1}s)(1+T_{p2}s)\ldots\ldots\ldots} = 0 \ ,$$

$$k_a = \frac{s^2 k(1)(1)\ldots}{s^1(1)(1)\ldots} = 0 \ ,$$

$$e_{ss} = \frac{A}{k_a} = \frac{A}{k} = \infty(\text{infinity}) \ ,$$

Hence when we subject a type 1 system to a parabolic input, the steady state error increases continuously. Hence type 1 system are not suitable when the input is parabolic in nature.



ParabolicInputtoaType2System:

we already know 'e_{ss}' for a parabolic input is

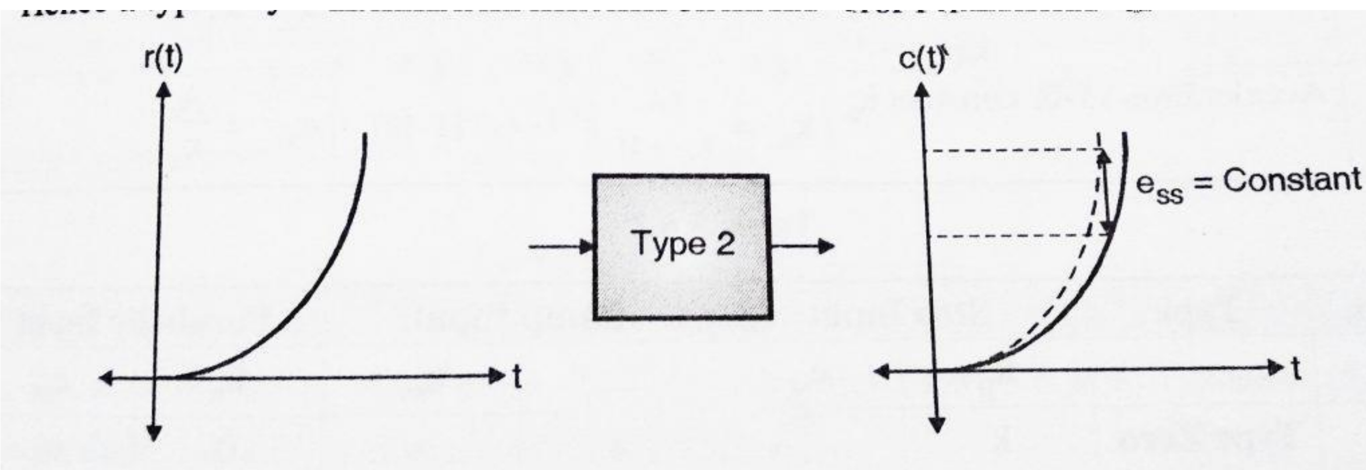
$e_{ss} = \frac{A}{k_a}$ where ,

$$k_a = \lim_{s \to 0}(s^2 G(s)H(s) = \lim_{s \to 0} \frac{s^2 k(1 + T_{z1}s)(1 + T_{z2}s)\ldots\ldots\ldots}{s^2(1 + T_{p1}s)(1 + T_{p2}s)\ldots\ldots\ldots}$$

$k_a = \frac{s^2 k(1)(1)\ldots.}{s^2(1)(1)\ldots.}$ = k ,

$e_{ss} = \frac{A}{k_a} = \frac{A}{k}$ = const ,

,



Hencewhenwesubjectatype2systemtoaparabolicinput,thesteadystateerrorisconstant.Hencewe can conclude that Type 2 systems are excellent for step and ramp signals and gives constant error for parabolic inputs.

Errorofdifferenttypesof input

Sr no	Type	Step input		Ramp input		Parabolic Input	
		k_p	e_{ss}	k_v	e_{ss}	k_a	e_{ss}
1	Type 0	k	$\frac{A}{1+k}$	0	∞	0	∞
2	Type 1	∞	0	k	$\frac{A}{k}$	0	∞
3	Type 2	∞	0	∞	0	k	$\frac{A}{k}$

5.6.Effect**ofaddingpolesandzerostothetransfer function**

To understand over damped, under damped and Critical damped in control system, Let we take the closed loop transferfunctioningenericform andanalysisthattofindoutdifferentconditionOverdamped,underdampedand Critical damped in control system.

$$T(s) = \frac{\omega_n^2}{s^2 + 2\delta\omega_n s + \omega_n^2}$$

NowweknowthatthetransientresponseofanysystemdependsonthepolesofthetransferfunctionT(s).Andas we know that the roots of the denominator polynomial in s of *T(s)* are the poles of the transfer function.
SoinourcasethedenominatorpolynomialofT(s),is

$$D(s) = s^2 + 2\delta\omega_n s + \omega_n^2$$

is known as the*characteristic polynomial*of the system and D(s) = 0 is known asthe*characteristic*equation ofthe system.

So ThepolesofT(s),or,the rootsofthecharacteristicequationwecanget by

$$s^2 + 2\,\delta\,\omega_n\,s + \omega_n^2 = 0$$

are given by,

$$s_{1,2} = \frac{-2\delta\omega_n \pm \sqrt{4\delta^2\omega_n^2 - 4\omega_n^2}}{2}$$

$$= -\delta\,\omega_n \pm j\,\omega_n\,\sqrt{1-\delta^2} \qquad \text{(assuming } \delta < 1\text{)}$$

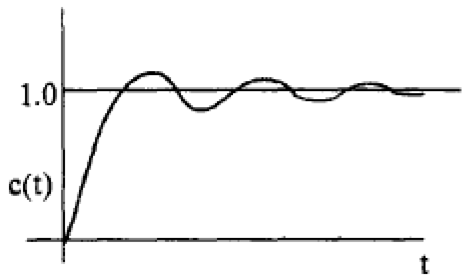
$$= -\delta\,\omega_n \pm j\,\omega_d$$

Where*is*knownasthe*dampednaturalfrequency*ofthe system.
NowIfδ>1,thetworootss$_1$ands$_2$arereal andwehaveanover dampedsystem. If $\delta = 1$, the system is known as a *critically damped system.*

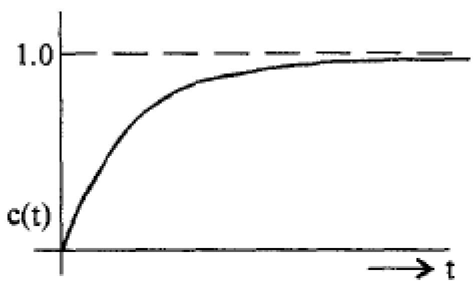
Themorecommoncaseof*0*<1isknownasthe*underdampedsystem.*

Nowifwegoforsteprespondsofdifferentsecondordersystemsthenwecan see

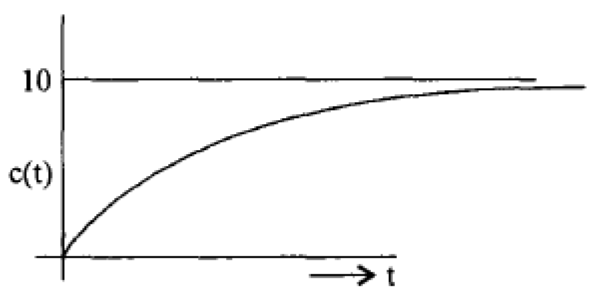
Stepresponseofanunderdampedsecondordersystem.



Stepresponseofacriticallydampedsecondorder system.



Stepresponseofanoverdampedsecondordersystem.



5.7.ResponsewithP,PI,PD,PIDcontroller

Processcontrolsarenecessaryfordesigningsafeandproductiveplants.Avarietyofprocesscontrolsareusedto manipulate processes, however the most simple and often most effective is the **PID controller**. The controller attempts to correct the error between a measured process variable and desired setpoint by calculating the differenceandthenperformingacorrectiveactiontoadjusttheprocessaccordingly.APIDcontrollercontrolsa process through three parameters: Proportional (P), Integral (I), and Derivative (D).

Proportional(P)Control

One type of action used in PID controllers is the proportional control. Proportional control is a form of feedback control. It is the simplest form of continuous control that can be used in a closed-looped system. P-only control minimizesthefluctuationintheprocessvariable,butitdoesnotalwaysbringthesystemtothedesiredsetpoint. It provides a faster response thanmost other controllers, initially allowing the P-only controller to respond a few secondsfaster.However, as the system becomes more complex (i.e.more complex algorithm) the response time difference could accumulate, allowing the P-controller to possibly respond even a few minutes faster. Although the P-only controller does offer the advantage of faster response time, it produces deviation from the set point. This deviation is known as the offset, and it is usually not desired in a process

P-controllinearlycorrelatesthecontrolleroutput(actuatingsignal)totheerror(diference betweenmeasured signal and set point). This P-control behavior is mathematically illustrated in Equation

$$c(t)=Kce(t)+b(9.2.2)(9.2.2)c(t)=Kce(t)+b$$

where

- c(t)c(t)=controlleroutput
- KcKc=controllergain
- e(t)e(t)= error
- bb= bias

Proportional-Integral(PI)Control

One combination is the **PI-control**, which lacks the D-control of the PID system. PI control is a form of feedback control. It provides a faster response time than I-only control due to the addition of the proportional action. PI control stops the system from fluctuating, and it is also able to return the system to its set point. Although the responsetimeforPI-controlisfasterthanI-onlycontrol,itisstillupto50%slowerthanP-onlycontrol.Therefore, in order to increase response time, PI control is often combined with D-only control.

PI-controlcorrelates thecontrolleroutputtotheerrorandtheintegraloftheerror.ThisPI-controlbehavioris mathematically illustrated in Equation

$$c(t)=Kc(e(t)+1Ti\int e(t)dt)+C(9.2.5)(9.2.5)c(t)=Kc(e(t)+1Ti\int e(t)dt)+C$$

where

- c(t)c(t)isthecontrolleroutput,
- KcKcisthecontroller gain,
- TiTiistheintegraltime,
- e(t)e(t)istheerror,and
- CCistheinitialvalueofcontroller

Inthisequation,theintegraltimeisthetimerequiredfortheI-onlyportionofthecontrollertomatchthecontrol provided by the P-only part of the controller.

TheequationindicatesthatthePI-controlleroperateslikeasimplifiedPID-controllerwithazeroderivativeterm. Alternatively,the PI-controller can also be seen as a combinationof the P-only and I-only control equations. The bias term in the P-only control is equal to the integral action of the I-only control. The P-only control is only in actionwhenthesystem isnotatthesetpoint.Whenthesystem isatthesetpoint,theerrorisequalto zero,and the first term drops out of the equation. The system is then being controlled only by the I-only portion of the controller. Should the system deviate from the set point again, P-only control will be enacted. A graphical representation of the PI-controller output for a step increase in input at time t0 is shown below in Figure 5. As expected, this graph resembles the qualitative combination of the P-only and I-only graphs.

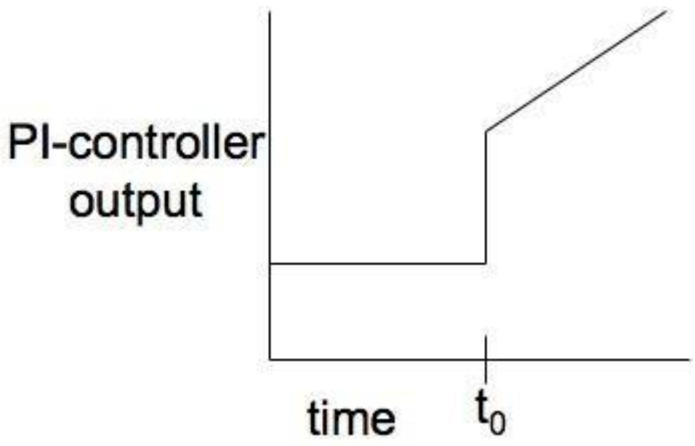


Figure.PI-controlleroutputforstepinput.

Proportional-Derivative(PD)Control

Another combination of controls is the PD-control, which lacks the I-control of the PID system. PD-control is combination of feedforward and feedback control, because it operates on both the current process conditions andpredictedprocessconditions.InPD-control,thecontroloutputisalinearcombinationoftheerrorsignaland its derivative. PD-control contains the proportional control's damping of the fluctuation and the derivative control'spredictionofprocesserror.

Asmentioned,PD-controlcorrelatesthecontrolleroutputtotheerrorandthederivativeoftheerror.ThisPD- control behavior is mathematically illustrated in Equation .

c(t)=Kc(e(t)+Tddedt)+C(9.2.6)(9.2.6)c(t)=Kc(e(t)+Tddedt)+C

where

- c(t)=controlleroutput
- K_c=proportionalgain
- e = error
- C=initialvalueof controller

The equation indicates that the PD-controller operateslike asimplified PID-controller with a zero integralterm. Alternatively,thePD-controllercanalso beseenasacombinationoftheP-onlyandD-onlycontrolequations.In this control, the purpose of the D-only control is to predict the error in order to increase stability of the closed loop system. P-D control is not commonly used because of the lack of the integral term. Without the integral term,theerrorinsteadystateoperationisnotminimized.P-DcontrolisusuallyusedinbatchpHcontrolloops, where error in steady state operation does not needto be minimized. In this application, the erroris relatedto the actuating signal both through the proportional and derivative term. A graphical representation of the PD-controller output for a step increase in input at time t0 is shown below in Figure 6. Again, this graph is a combination of the P-only and D-only graphs, as expected.

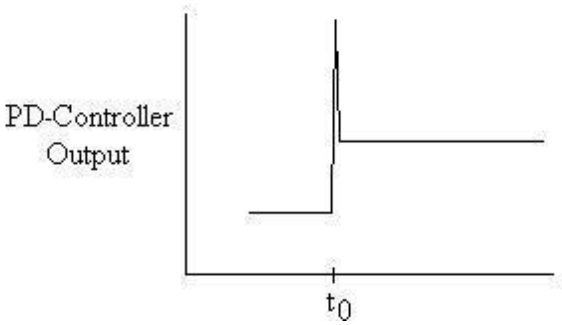


Figure.PD-controlleroutputforstepinput.

Proportional-Integral-Derivative(PID)Control

Proportional-integral-derivativecontrolisacombinationofallthreetypesofcontrolmethods.PID-controlismost commonly used because it combines the advantages of each type of control. This includes a quicker responsetime because of the P-only control, along with the decreased/zero offset from the combined derivative and integral controllers. This offset was removed by additionally using the I-control. The addition of D-control greatly increases the controller's response when used in combination because it predicts disturbances to the system by measuring the change in error. On the contrary, as mentioned previously, when used individually, it has a slower response time compared to the quicker P-only control. However, although the PID controller seems to be the most adequate controller, it is also the most expensive controller. Therefore, it is not used unless the process requires the accuracy and stability provided by the PID controller.

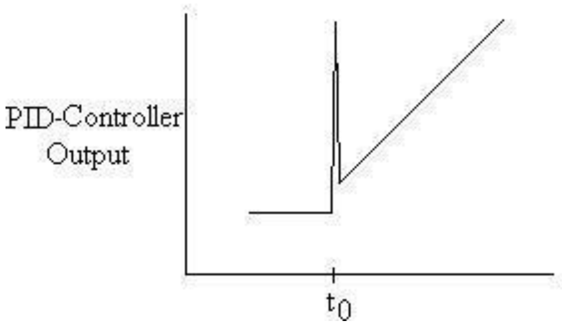
PID-controlcorrelatesthecontrolleroutputtotheerror,integraloftheerror,andderivativeoftheerror.ThisPID- control behavior is mathematically illustrated in Equation 6 (Scrcek, *et. al*).

$$c(t)=Kc(e(t)+1Ti\int e(t)dt+Tddedt)+C(9.2.7)(9.2.7)c(t)=Kc(e(t)+1Ti\int e(t)dt+Tddedt)+C$$

where

- c(t)=controlleroutput
- K_c=controllergain
- e(t)=error
- T_i=integraltime
- T_d=derivativetimeconstant
- C=intitialvalueofcontroller

As shownin the above equation, PID controlis the combinationof all three types of control. In thisequation, the gain is multiplied with the integral and derivative terms, along with the proportional term, because in PID combination control, the gain affects the I and D actions as well. Because of the use of derivative control, PID control cannot be used in processes where there is a lot of noise, since the noise would interfere with the predictive, feedforward aspect. However, PID control is used when the process requires no offset and a fast response time. A graphical representation of the PID-controller output for a step increase in input at time t0 is shownbelowinFigure.ThisgraphresemblesthequalitativecombinationoftheP-only,I-only,andD-onlygraphs.



FigurePID-controlleroutputforstepinput.

ShortQuestions

1. Whatisanorderofa system?

Theorderofasystemistheorderofthedifferentialequationgoverningthesystem.Theorderofthesystem can be obtained from the transfer function of the given system.

2. Whatisstepsignal?

ThestepsignalisasignalwhosevaluechangesfromzerotoAatt=0andremainsconstantatAfort>0.

3. Whatisramp signal?

The ramp signal is a signal whose value increases linearly with time from aninitial value of zero at t=0.the ramp signal resembles a constant velocity.

4. Whatisaparabolic signal?

Theparabolicsignalisasignalwhosevaluevariesasasquareoftimefromaninitialvalueofzeroatt=0.This parabolic signal represents constant acceleration input to the signal.

5. **Whatistransientresponse?**

Thetransientresponseistheresponseofthesystemwhenthesystemchangesfromonestatetoanother.

6. **Whatissteadystateresponse?**

Thesteadystateresponseistheresponseofthesystemwhenitapproachesinfinity.

7. **DefineDampingratio.**

Dampingratioisdefinedastheratioofactualdampingtocritical Damping.

8. **Listthetimedomainspecifications.**

Thetimedomainspecificationsare

i. Delaytime

ii. Rise time

iii. Peaktime

iv. Peakovershoot

9. **Whatisdampedfrequencyofoscillation?**

Inunderdampedsystemthe responseisdampedoscillatory.Thefrequencyofdampedoscillationisgiven by$\omega_d=\omega_n\sqrt{(1-\zeta^2)}$

10. Whatwillbethenatureofresponseofsecondordersystemwithdifferenttypesofdamping?

Forundampedsystemtheresponseisoscillatory.

Forunderdampedsystemtheresponseisdampedoscillatory.

Forcriticallydampedsystemtheresponseisexponentiallyrising.

Foroverdampedsystemtheresponseisexponentiallyrisingbuttherisetimewillbevery large.

11. DefineDelaytime.

Thetimetakenforresponsetoreach50%offinalvalueforttheveryfirsttimeis delay time.

12. DefineRise time.

Thetimetakenforresponsetoraisefrom0%to100%fortheveryfirsttimeisrise time.

13. Definepeak time

Thetimetakenfortheresponsetoreachthepeakvalueforthefirsttimeispeaktime.

14. Definepeakovershoot.

PeakovershootisdefinedastheratioofmaximumpeakvaluemeasuredfromtheMaximumvaluetofinal value

15. DefineSettlingtime.

Settlingtimeisdefinedasthetimetakenbytheresponsetoreachandstaywithinspecifiederror

16. Whatistheneedforacontroller?

Thecontrollerisprovidedtomodifytheerrorsignalforbettercontrolaction.

17. Whatarethedifferenttypesofcontrollers?

Thedifferenttypesofthecontrollerare

Proportionalcontroller

PI controller

PD controller

PIDcontroller

18. Whatisproportionalcontroller?

Itisdevicethatproducesacontrolsignalwhichis proportionaltotheinputerror signal.

19. WhatisPIcontroller?

Itisdevicethatproducesacontrolsignalconsistingoftwoterms –oneproportionaltoerrorsignalandthe other proportional to the integral of error signal.

20. WhatisPD controller?

PDcontrollerisaproportionalplusderivativecontrollerwhichproducesanoutputsignalconsistingoftwo terms - one proportional to error signal and other proportional to the derivative of the signal.

21. Whatisthesignificanceofintegralcontrollerandderivativecontrollerina PIDcontroller?

Theproportionalcontrollerstabilizesthegainbutproducesasteadystateerror.Theintegralcontrol reduces or eliminates the steady state error.

22. DefineSteadystate error.

Thesteadystateerroristhevalueoferrorsignale(t)whenttendstoinfinity.

23. Whatisthedrawbackofstaticcoefficients?

The main drawback of static coefficient is that it does not show the variation of error with time and input should be standard input.

24. Whatarethethreeconstantsassociatedwithasteadystateerror?

Thethreesteadystateerrorsconstantare

Positional error constant

K_pVelocity error constant

K_vAccelerationerrorconstantK$_a$

25. Whatarethemainadvantagesofgeneralizederrorco-efficients?

i) Steadystateisfunctionof time.

ii) Steadystatecanbedeterminedfromanytypeofinput.

26. Whataretheeffectsofaddingazerotoa system?

Addingazerotoasystemresultsinpronouncedearlypeaktosystemresponsetherebythepeakovershoot increases appreciably.

27. Whyderivativecontrollerisnotusedincontrolsystem?

The derivative controller produces a control action based on rate of change of error signal and it does not produce corrective measures for any constant error. Hence derivative controller is not used in control system

28. WhatistheeffectofPIcontrolleronthesystemperformance?

ThePIcontrollerincreasestheorderofthesystembyone,whichresultsinreducingthesteadystateerror .Butthesystembecomeslessstablethantheoriginalsystem.

29. WhatistheeffectofPDcontrolleronsystemperformance?

TheeffectofPDcontrolleristoincreasethedampingratioofthesystemandsothepeakovershootis reduced.

30. Whatisthedisadvantageinproportionalcontroller?

Thedisadvantageinproportionalcontrolleristhatitproducesaconstantsteadystateerror.

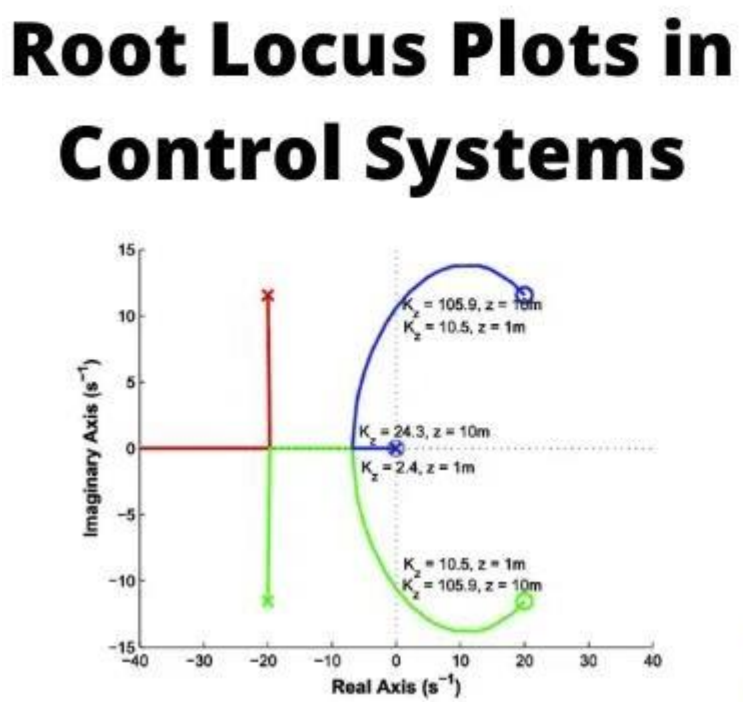
Long Questions

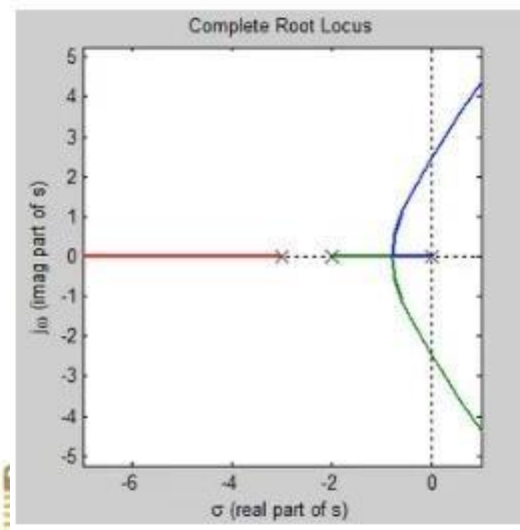
1. Derivetimeresponseoffirstordersystemwithunitstepresponse?
2. explaindifferenttypesofcontroller?
3. Deriveexpressionforrisetime,peaktime,peakovershoot,?

Chapter-6

AnalysisofStabilityByrootLocusTechinique

6.1. RootLocusconcept





The**rootlocustechniqueincontrolsystem**wasfirstintroducedintheyear1948byEvans.Anyphysicalsystemis represented by a transfer function in the form of

$$G(s) = k \times \frac{numerator\ of\ s}{denomerator\ of\ s}$$

We canfindpolesandzeros from G(s). The locationofpolesandzeros arecrucialkeepingview stability, relative stability, transient response and error analysis. When the system is put to service stray inductanceand capacitancegetintothesystem,thuschangesthelocationofpolesandzeros.In **rootlocus techniqueincontrol system** we will evaluate the position of the roots, their locus of movement and associated information. These information will be used to comment upon the system performance.
NowbeforeIintroducewhatisarootlocustechnique,itisveryessentialheretodiscussafewoftheadvantages of thistechniqueoverother stabilitycriteria. Someofthe advantagesof root locus technique are written below.

Advantages of Root Locus Technique

1. Rootlocustechniqueincontrolsystemiseasytoimplementascomparedtoothermethods.
2. Withthehelpofrootlocuswecaneasilypredicttheperformanceofthewhole system.
3. Rootlocusprovidesthebetterwaytoindicatethe parameters.

Nowtherearevarioustermsrelatedtorootlocustechniquethatwewillusefrequentlyinthis article.

1. Characteristic Equation Related to Root Locus Technique : 1 + G(s)H(s) = 0 is known as characteristicequation.Nowondifferentiatingthecharacteristicequationandonequatingdk/ds equals to zero, we can get break away points.

2. BreakawayPoints:Supposetworootlociwhichstartfrompoleandmovesinoppositedirection collide with each other such that after collision they start moving in different directions in the symmetricalway.Orthebreakawaypoints atwhichmultiplerootsofthecharacteristic equation 1 + G(s)H(s) = 0 occur. The value of K is maximum at the points where the branches of root loci break away. Break away points may be real, imaginary or complex.
3. BreakinPoint :Conditionofbreakintobethereontheplotiswrittenbelow: Root locusmustbe present between two adjacent zeros on the real axis.
4. Centre of Gravity : It is also known centroid and is defined as the point on the plot from where all theasymptotesstart.Mathematically,itiscalculatedbythedifferenceofsummationofpolesand zeros in the transfer function when divided by the difference of total number of poles and total number of zeros. Centre of gravity is always real and it is denoted by σ_A.

$$\sigma_A = \frac{(Sum\ of\ real\ parts\ of\ poles) - (Sum\ of\ real\ parts\ of\ zeros)}{N - M}$$

   Where,NisnumberofpolesandMisnumberofzeros.
5. Asymptotesof RootLoci:Asymptoteoriginatesfromthecenterofgravityorcentroidandgoesto infinity at definite some angle. Asymptotes provide direction to the root locus when they depart break away points.
6. AngleofAsymptotes:Asymptotesmakessomeanglewiththe realaxisandthisanglecanbe calculated from the given formula,

$$Angle\ of\ asymptotes\ = \frac{(2p+1) \times 180}{N - M}$$

   Where,p=0,1,2…….(N-M-1) N is
   the total number of poles
   Misthetotalnumberofzeros.
7. Angle of Arrival or Departure : We calculate angle of departure when there exists complex polesinthesystem.Angleofdeparturecanbecalculatedas180-{(sumofanglestoacomplexpolefrom the other poles)-(sum of angle to a complex pole from the zeros)}.
8. Intersection of Root Locus with the Imaginary Axis : In order to find out the point of intersection rootlocuswithimaginaryaxis,wehavetouseRouthHurwitzcriterion.First,wefindtheauxiliary equation then the corresponding value of K will give the value of the point of intersection.
9. Gain Margin : We define gain margin by which the design value of the gain factor can be multipliedbeforethesystembecomesunstable.Mathematicallyitisgivenbytheformula

$$Gain\ margin = \frac{Value\ of\ K\ at\ the\ imaginary\ axes\ cross\ over}{Design\ value\ of\ K}$$

10. PhaseMargin:Phasemargincanbecalculatedfromthegivenformula:

$$Phase\ margin = 180 + \angle(G(jw)H(jw))$$

11. SymmetryofRootLocus:Rootlocusissymmetricabout thexaxisortherealaxis.

HowtodeterminethevalueofKatanypointontherootloci?Nowtherearetwo waysofdeterminingthevalue of K, each way is described below.

1. MagnitudeCriteria:Atanypointsontherootlocuswecanapplymagnitudecriteriaas,

   $$|G(s)H(s)| = 1$$

   Usingthisformulawecancalculatethevalue ofKatanydesired point.
2. UsingRootLocusPlot :ThevalueofKatanysontherootlocusisgivenby

$$K = \frac{product\ of\ all\ of\ the\ vector\ lengths\ drawn\ from\ the\ poles\ of\ G(s)H(s)\ to\ s}{product\ of\ all\ of\ the\ vector\ lengths\ drawn\ from\ the\ zeros\ of\ G(s)H(s)\ to\ s}$$

6.2. ConstructionofRootLocus Plot

This is also known as root locus technique in control system and is used for determining the stability ofthe given system.Nowinordertodeterminethestabilityofthesystemusingtherootlocustechniquewefindtherangeof values of K for which the complete performance of the system will be satisfactory and the operation is stable. Nowtherearesomeresultsthatoneshouldrememberinordertoplottherootlocus.Theseresultsarewritten below:

1. Region where root locus exists : After plotting all the poles and zeros onthe plane, we can easily findouttheregionofexistenceoftherootlocusbyusingonesimplerulewhichiswrittenbelow, Only that segment will be considered in making root locus if the total number ofpoles and zeros at the right hand side of the segment is odd.
2. Howtocalculate thenumber ofseparate root loci? :Anumberofseparaterootlociareequalto the total number of roots if number of roots are greater than the number of poles otherwise numberof separateroot loci is equalto thetotal number of polesif numberof roots are greater than the number of zeros.

6.3. RulestoPlotRootLocus

Keepingallthesepointsinmindweareableto draw the **root locus plot**foranykindofsystem.Now letusdiscuss the procedure of making a root locus.

1. Findoutalltheroots andpolesfromtheopenlooptransferfunctionandthenplotthemonthe complex plane.
2. Alltherootlocistartsfromthepoleswherek=0andterminatesatthezeroswhereKtendsto infinity. The number of branches terminating at infinity equals to the difference between the number of poles & number of zeros of G(s)H(s).
3. Findtheregionofexistenceoftherootlocifromthemethoddescribedaboveafterfindingthe values of M and N.
4. Calculatebreakawaypointsandbreakinpointsifany.
5. Plottheasymptotesandcentroidpointonthecomplexplanefortherootlocibycalculatingthe slope of the asymptotes.
6. Nowcalculateangleofdepartureandtheintersectionofrootlociwithimaginaryaxis.
7. NowdeterminethevalueofKbyusinganyonemethodthatIhavedescribed above. Byfollowingaboveprocedureyoucaneasilydrawthe**rootlocusplot**foranyopenlooptransfer function.
8. Calculatethegainmargin.
9. Calculatethephasemargin.
10. YoucaneasilycommentonthestabilityofthesystembyusingRouth Array.

EffectsofAddingOpenLoopPolesandZerosonRootLocus

Therootlocuscanbeshiftedin **'s'plane**byaddingtheopenlooppolesandtheopenloopzeros.

- Ifweincludeapoleintheopenlooptransferfunction,thensomeofrootlocusbrancheswillmove towards right halfof 's' plane. Because of this, the damping ratio δδ decreases. Which implies, dampedfrequencyωdωdincreasesandthe time domain specifications like delay timetdtd, rise time trtr andpeaktime tptp decrease.But,iteffectsthe

  systemstability.
- Ifweincludeazerointheopenlooptransferfunction,thensomeofrootlocusbrancheswillmove towards left half of 's' plane. So, it will increase the control system stability. In this case, the damping ratio δδincreases. Which implies, damped frequency ωdωd decreases and the time domain specifications like delay time tdtd, rise time trtr and peak time tptp increase.

So,basedontherequirement,wecaninclude(add)theopenlooppolesorzerostothetransferfunction.

Short questions

1. ConsiderthelooptransferfunctionK(s+6)/(s+3)(s+5)findout-Intherootlocusdiagramthecentroidwill be located at________________?

Ans-Centroid=Sumofrealpartofopenloop pole-sumofrealpartofopenloopzeros/P-Z.

2. Whatisthenumberoftherootlocussegmentswhichdonotterminateonzeroes?

Ans-Thenumberoftherootlocussegmentswhichdo notlieontherootlocusisthedifferencebetweenthe number of the poles and zeroes.

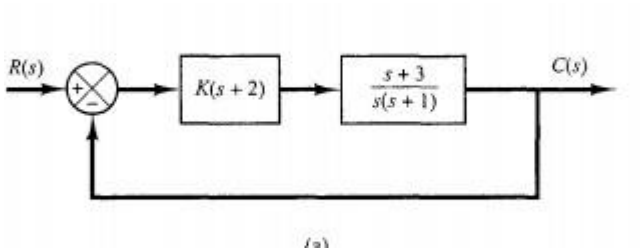
3. Ifthegainofthesystemisreducedtoazerovalue,therootsofthesysteminthes-plane,
Ans-Therootsofthesystem insplanecoincideswiththepoles ifthegainofthesystemisreducedto avalue zero.

4. Whenthenumberofpolesisequaltothenumberofzeroes,howmanybranchesofrootlocustendstowards infinity?

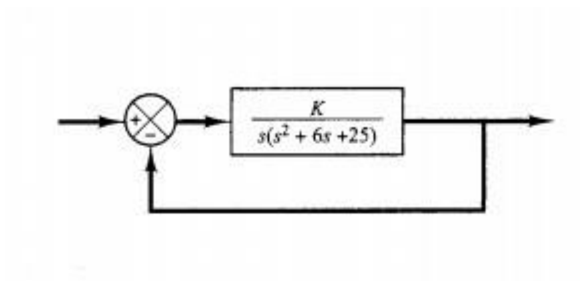
Ans:Branchesoftherootlocusisequaltothenumberofpolesorzeroeswhichever isgreaterandtendstoward infinity when poles or zeroes are unequal.

Long question:

1. SketchtherootlociforthesystemshowninFigure).(ThegainKisassumedtobepositive.)Observethatfor small or large values of K the system is overdamped and for medium values of K it is underdamped.



2. SketchtherootlociofthecontrolsystemshowninFigure



Chapter-7
FrequencyresponseAnalysis

7.1. Corelationbetweentimeresponseandfrequencyresponse

As has been stated, the use of frequency response for the design of control systems requires a Correlation Between Frequency and Transient Response. Time response specifications are availablefortheperformanceofasystem.Thesemustbetranslatedtofrequencyresponse.There must be frequency domain specifications also, corresponding to the time domain specifications, such as overshoot, settling time, etc. However, it is easy to have a direct Correlation Between FrequencyandTransientResponseofasecondordersystem.Atypicalmagnitudeplotofasecond order system is shown in Fig. 6.17.

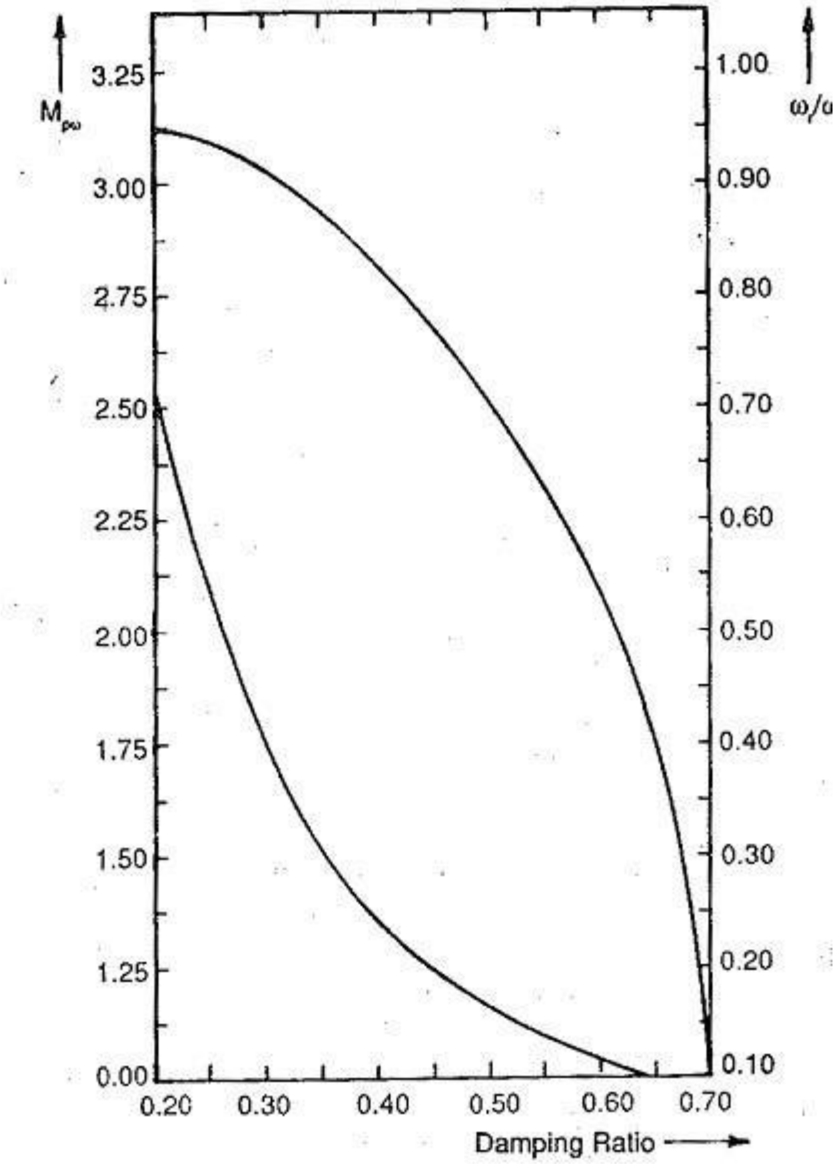


Fig. 6.18 *Peak of frequency response and resonating frequency of second order system as a function of damping ratio*

Theresonantpeak(maximumamplitude)$M_{P\omega}$ofthemagnitudeplotdependsuponthedamping ratioofthesystem.Theresonantfrequencyalsodependsuponthedampingratio.Theserelations are given by Eqs 6.59 and 6.60 and represented graphically in Fig. 6.18. The resonant frequency $\omega_r$andbandwidthofthefrequencyresponserelatetothefastnessofresponse.Smallerthevalues ofbandwidthandresonantfrequencysmalleristherisetimeofthetransientresponseandfaster is the response. The overshoot of the time response can be related to the resonant peak of the frequency response. This resonant peak also indicates the relative stabilityof the systems. The bandwidth is related to the natural frequency $\omega_n$of the system. For a given ξ greater the value $\omega_n$faster is the response. The value of ξ must be chosen to compromise between $M_{p\omega}$and ω_r.

Thefrequencydomain specificationsaretherefore

1. Thepeakamplitude andthefrequencyatwhichthis occurs.Thepeakamplitudemust be normally less than 1.5. The acceptable range of peak amplitude corresponds to damping rates of 0.4 to 0.7.

2. Relatively large resonant frequency and hence large bandwidth of the frequency response. The systemwillhave relatively smalltime constants.The system becomesfaster.

3. The closeness of the polar plotof frequency response to (-1, 0) point indicates the peak overshoot of the time response. This also gives the relative

4. Thesteady-stateerrorcanberelatedtofrequencyresponsealso.Thegainandnumberof integrations involved in the open loop systemindicate the influence of steady-state error.

The Correlation Between Frequency and Transient Response of higher order systems is not so simple and straightforward as it is for second order systems. The mathematical treatment of higher order systems for such a correlation is rather involved and laborious. However, a higher order system can be represented by a second order system if it has a pair of dominant complexconjugate poles. The frequency and time responsesofthissystemisinfluencedbythispairofdominantpoles.InsuchacasetheCorrelationBetween Frequency and Transient Responseforasecond ordersystem can bevery easily extended forhigher order systems.

For higher order systems having a dominant pair of complex conjugate poles, the following correlation exists between the transient and frequency responses:

1. Thepeak magnitudeoffrequency responseindicates therelativestability.Asystem havinga peak amplitudein therange of1 to 1.4 would havea timeresponsewith an effective damping ratioin the range of 0.4 to 0.7.

2. Ifthepeakamplitudeofthefrequencyresponseisgreaterthan1.5thetimeresponseisoscillatory, having a large overshoot.

3. The resonant frequency, i.e. the frequency corresponding to peak amplitude, is a measure of the fastnessofresponse.Largerthevalueofresonantfrequency,fasteristheresponse,i.e.thesmalleris the rise time.

4. Thesystemishighlydampedifttheresonantfrequencyanddampednaturalfrequencyarecloseto each other.

5. Larger values of to ω_r characterizelarger bandwidth. However, in view of the noise the system should not have largebandwidth. Larger the bandwidth costlier is the system. A compromise is required.

6. Cut off frequency (frequency at which the amplitude is 3 db below the zero frequency value) characterises the filtering characteristics of high frequency

7. The slope of the log-magnitude curves known as cut off rate gives the ability of the control system to distinguish between noise and signal.

Using the above correlation, the time domain specifications can be translated to frequency domain specifications.Thedesignofthecontrolsystemiscarriedoutinthefrequencydomaintomeettherequired specifications.

7.2. PolarPlot

Definition:Theplotthatrepresentsthetransferfunctionofthesystem $G(j\omega)$ona complex plane, constructed in polar coordinates is known as Polar Plot.

Thepolarplotrepresentationshowstheplotofmagnitudeversusphaseangleon polar coordinates with variation in ω from 0 to ∞. It is used for stability analysis.

ConstructionofPolarPlot

Itisknowntousthatplottingfrequencyresponsesignifiessketchingthevariationsinthe magnitude and phase angle with respect to the input frequency. These plots are known as magnitude plot (gain plot) and phase plot respectively.

IntheBodeplot,thefrequencyresponseissketchedusingalogarithmicscale.

So,inapolarplot,asketchbetweenthemagnitudeandphaseangleofthetransfer function $G(j\omega)$ is formed for different values of ω.

SupposeMrepresentsthemagnitudeandφdenotesthephaseangle,thenforthe transfer function of a system it is given as:

$$M = | G(j\omega) H(j\omega) |$$

$$\phi = \angle G(j\omega) H(j\omega)$$

So,withthevariationinωfrom0to∞,thevaluesofMandφcanbedetermined.

Aswehavealreadydiscussedinthebeginningthatpolarplotismagnitudeversus phase angle graph plotted for various values of ω.

So,toconstructapolarplot,thedifferentvaluesofmagnitudeandphaseangleis tabulated and further, the sketch is formed. The table is given below:

Frequency	Magnitude	Phase Angle
0	M_0	φ_0
ω_1	M_1	φ_1
ω_2	M_2	φ_2
|	|	|
|	|	|
∞	M_∞	φ_∞

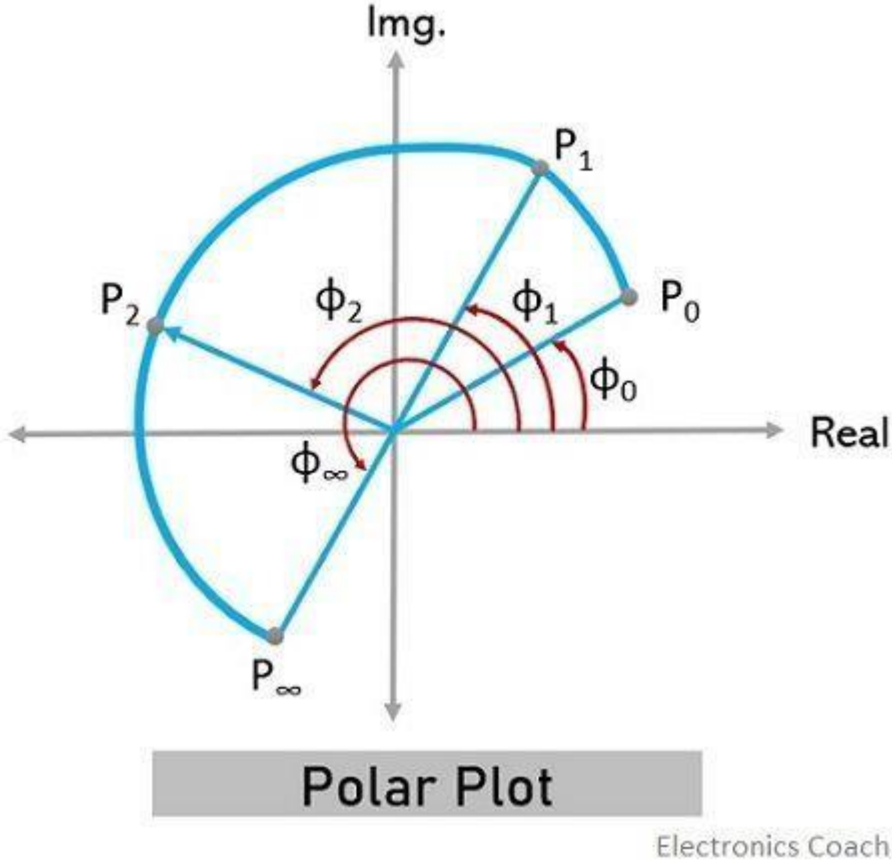
Basically,hereeachpointonthepolarplotissignificantlyplottedforeachspecificvalue of magnitude and phase angle for particular frequency ω.

Likefromtheabovetable,for$\omega=\omega_1$,$M=M_1$and$\varphi=\varphi_1$apointinthepolarcoordinate systemisdecidedthatrepresents$M_1\angle\varphi_1$,hence,thepointontheplotcorrespondstothe tip of the phasor of magnitude M_1plotted at an angle φ_1.

So, by using the tabulated data, the polar plot can be formed. Thus, in this way, the magnitudevsphaseangleplotiscanbeconstructedforvariousvaluesoffrequency.

It is to be noted here that conversion of magnitude into dB or logarithm values is not necessary.Also,theanticlockwisedirectionrepresentspositivephaseangles,whilethe clockwise direction shows the negative phase angles.

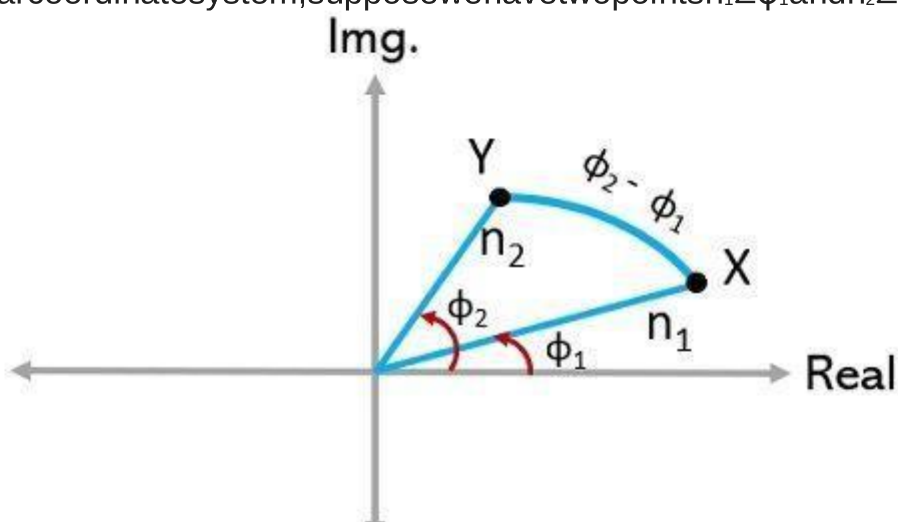
Thefigurebelowrepresentsthepolarplotforωbetween0to ∞:



Thus,fromtheabovediscussion,weconcludethatpolarplotisstartedfroma point specifying magnitude and angle for ω = 0 and is terminated at a point specifying magnitude and angle for ω = ∞.

- Anothermethodisusedtoroughlysketchapolarplotinwhichmagnitude and angles for the various values of ω are not calculated.

Basically,inapolarcoordinatesystem,supposewehavetwopointsn$_1$∠φ$_1$andn$_2$∠φ$_2$as



indicated below:

Here,itisclearfromtheabovefigurethatmovementofpointXfromY,causesanangle rotation,$\varphi_2 – \varphi_1$.Andifthedifferenceisnegative,therotationwillbeintheclockwise direction. While, if the difference is positive, the rotation will be in the anti-clockwise direction.

Inasimilarway,thevariationinωfrom0to∞,twopointscanbeconsidered.Oneatω= 0,withmagnitudeM_0andangle$\varphi_0$whiletheotheratω=∞withmagnitudeM_∞andangle φ_∞.Thentherewillbearotationfromφ_∞toφ_0.

Moresimply,

ω=0gives**$M_0\angle\varphi_0$**isthestartingpoint,

ω=∞gives**$M_\infty\angle\varphi_\infty$**istheterminatingpointand

$\varphi_\infty – \varphi_0$correspondstotherotation

Hence,inthisway,thepolarplotcanbeconstructed.

ExampleofPolarPlot

Tillnow,wehavediscussedwhatbasicallyapolarplotisandhowitisconstructedletus now consider an example to understand the construction of polar plot in a better way.

SupposewehaveaType0systemwhosetransferfunctionisgivenas: We have $G(s) = \frac{1}{1+s}$ to sketch the polar plot for it.

Thefirststepistoconvertthegiventransferfunctionintothefrequencydomain.Thus,it will be written as:

$$G(j\omega) = \frac{1}{1+j\omega}$$

$$G(j\omega) = \frac{1+j0}{1+j\omega}$$

Now,furthercalculatingthemagnitude, $| G(j\omega)\, H(j\omega) | = M = \frac{1}{\sqrt{1+\omega^2}}$

$$\angle G(j\omega)\,H(j\omega) = \phi = \frac{tan^{-1}(\frac{0}{1})}{tan^{-1}(\frac{\omega}{1})}$$

$$\angle G(j\omega)\,H(j\omega) = \phi = \frac{0^\circ}{(tan^{-1}\omega)}$$

Also,thephaseanglecondition, $\angle G(j\omega)\,H(j\omega) = \phi = -\,tan^{-1}(\omega)$

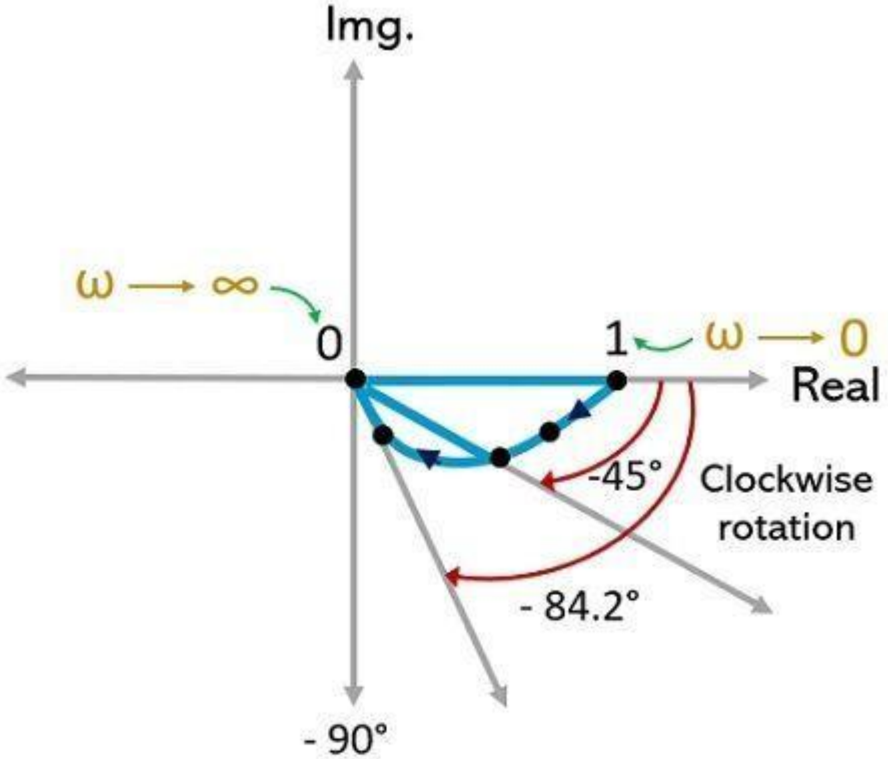
Now,wehavetocalculatemagnitudeandanglebysubstitutingdifferentvaluesofω between 0 and ∞.

Thus,thetabularrepresentationwillbe:

Frequency	Magnitude	Phase Angle
0	1	0°
1	$1/\sqrt{2}$	-45°
10	$1/\sqrt{101}$	-84.2°
|	|	|
|	|	|
∞	0	-90°

Hence,thetabulateddatashowsthatthestartingpointis1 ∠0°andterminatingpointis 0 ∠-90°. Thus, the plot will terminate at the origin, tangential to the axis of angle -90°.

Thus,theplotisrepresentedas:



Now,letusapplythealternativemethodtosketchthepolarplot.

Aswehavediscussedearlierthatinthismethodonlythestartingandterminatingpoints are of major significance. Thus, frequency is needed for 0 and ∞.

Fromtheabovetabularrepresentation,itisclearthat, For, **ω**

= 0 magnitude and angle = **1** ∠**0°**

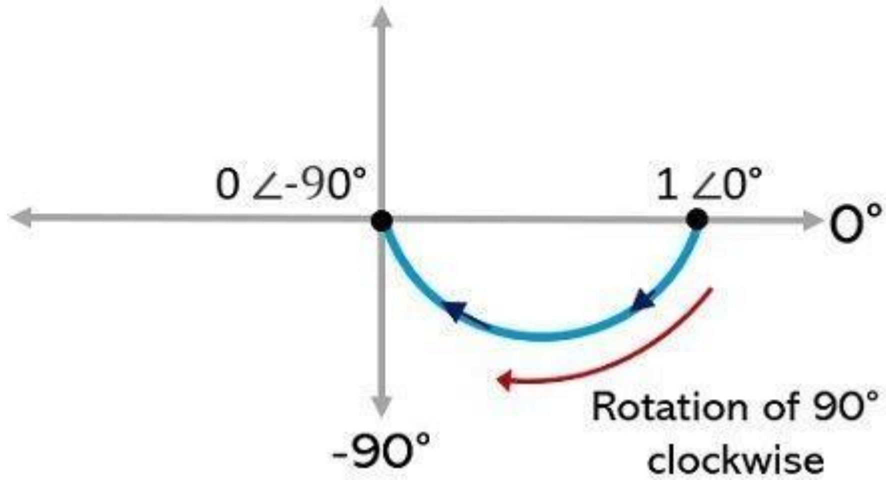
For,**ω=∞**magnitudeandangle=**0**∠**-90°**

Therefore,

$\varphi_\infty – \varphi_0$=-90°–0°=–90°

Asthedifferenceofthetwoisnegative,thus,therotationfrom startingtotheterminating point will be in the clockwise direction.

Thus,thestartingpoint,1∠0°isrotated90°intheclockwisedirection,inordertoget terminated at 0 ∠-90°. Hence, the rough sketch of the polar plot is given below:



Itistobenotedherethatmostlythisapproximatemethodisusedforsketchingthepolar plot.

7.3. BodePlot

TheBodeplotortheBodediagramconsistsoftwoplots−

- Magnitudeplot
- Phaseplot

In both the plots, x-axis represents angular frequency (logarithmic scale). Whereas, yaxis representsthemagnitude(linearscale)ofopenlooptransferfunctioninthemagnitudeplotand the phase angle (linear scale) of the open loop transfer function in the phase plot.

The**magnitude**oftheopenlooptransferfunctionindBis-

$$M=20\log|G(j\omega)H(j\omega)| \quad M=20\log|G(j\omega)H(j\omega)|$$

The**phaseangle**oftheopenlooptransferfunctionindegreesis-

$$\varphi=\angle G(j\omega)H(j\omega) \quad \varphi=\angle G(j\omega)H(j\omega)$$

Note−Thebaseoflogarithmis10.

BasicofBodePlots

Thefollowingtableshowstheslope,magnitudeandthephaseanglevaluesofthetermspresent in the open loop transfer function. This data is useful while drawing the Bode plots.

Type of term	G(jω)H(jω)	Slope(dB /dec)	Magnitude(dB)	Phase angle(degrees)
Constant	KK	$0$0	$20\log K$20log K	$0$0
Zero at origin	$j\omega$jω	$20$20	$20\log\omega$20log ω	$90$90
‘n’ zeros at origin	$(j\omega)^n$(jω)n	$20n$20n	$20n\log\omega$20nlog ω	$90n$90n
Pole at	1jω1jω	-20−20	$-20\log\omega$−20log ω	$-90 or 270$−90or270

origin				
'n' poles at origin	1(jω)n1(jω)n	−20n−20n	−20nlogω−20nlogω	−90nor270n−90nor270n
Simplezero	1+jωr1+jωr	2020	0forω<1r0forω<1r 20logωrforω>1r20logωrforω>1r	0forω<1r0forω<1r 90forω>1r90forω>1r
Simplepole	11+jωr11+jωr	−20−20	0forω<1r0forω<1r −20logωrforω>1r−20logωrforω>1r	0forω<1r0forω<1r −90or270forω>1r−90or270forω>1r
Second order derivativeterm	ω2n(1−ω2ω2n+2jδωωn)ωn2(1−ω2ωn2+2jδωωn)	4040	40logωnforω<ωn40logωnforω<ωn 20log(2δω2n)forω=ωn20log(2δωn2)forω=ωn 40logωforω>ωn40logωforω>ωn	0forω<ωn0forω<ωn90forω=ωn90forω=ωn180forω>ωn180forω>ωn
Second order integralterm	1ω2n(1−ω2ω2n+2jδωωn)1ωn2(1−ω2ωn2+2jδωωn)	−40−40	−40logωnforω<ωn−40logωnforω<ωn −20log(2δω2n)forω=ωn−20log(2δωn2)forω=ωn −40logωforω>ωn−40logωforω>ωn	−0forω<ωn−0forω<ωn −90forω=ωn−90forω=ωn −180forω>ωn−180forω>ωn

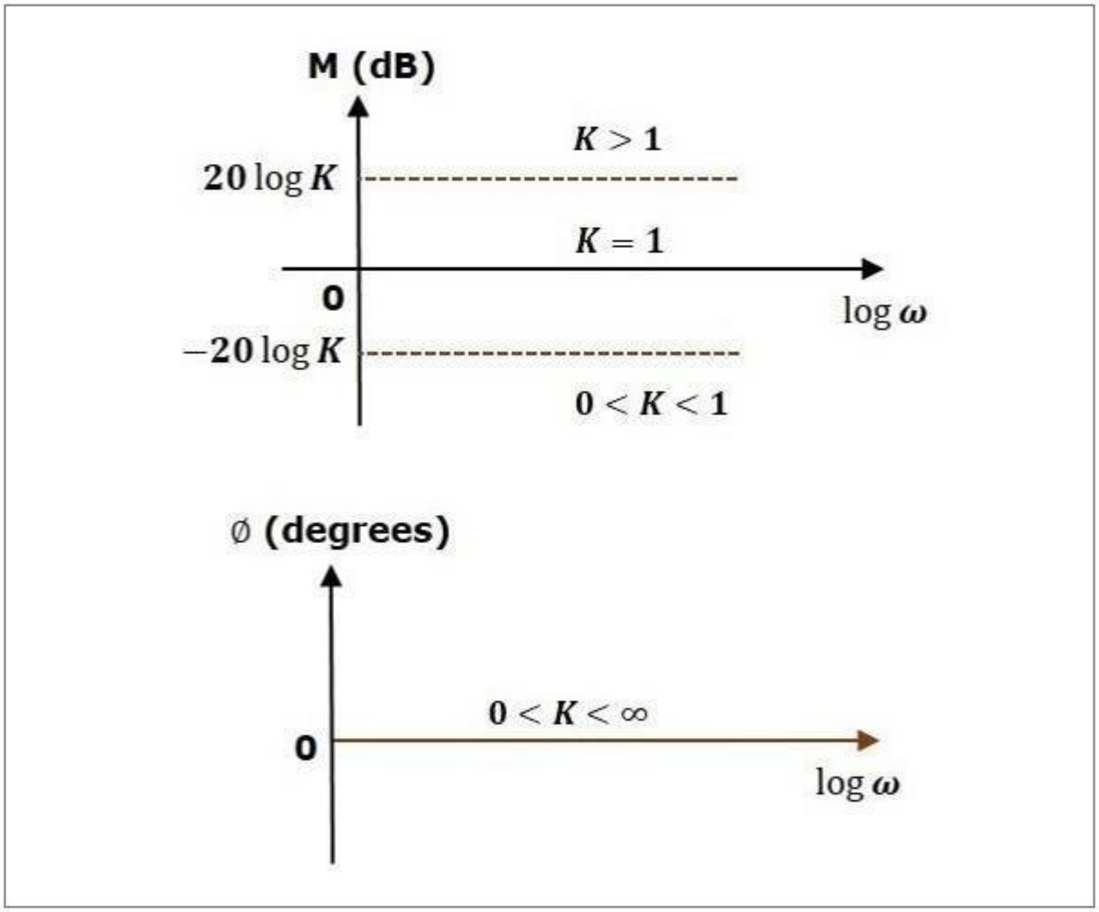
ConsidertheopenlooptransferfunctionG(s)H(s)=KG(s)H(s)=K.
Magnitude M=20logKM=20logK dB

Phaseangleφ=0φ=0degrees

IfK=1K=1,thenmagnitudeis0dB.

IfK>1K>1,thenmagnitudewillbepositive.If K<1K<1, then magnitude will be negative.

ThefollowingfigureshowsthecorrespondingBodeplot.



Themagnitudeplotisahorizontalline,whichisindependentoffrequency.The0dBlineitselfis the magnitude plot when the value of K is one. For the positive values of K,the horizontal line willshift20logK20log⁡KdBabovethe0dBline.Forthenegativevalues of K,thehorizontalline willshift20logK20log⁡KdBbelowthe0dBline.The Zerodegreeslineitselfisthephaseplotfor all the positive values of K.

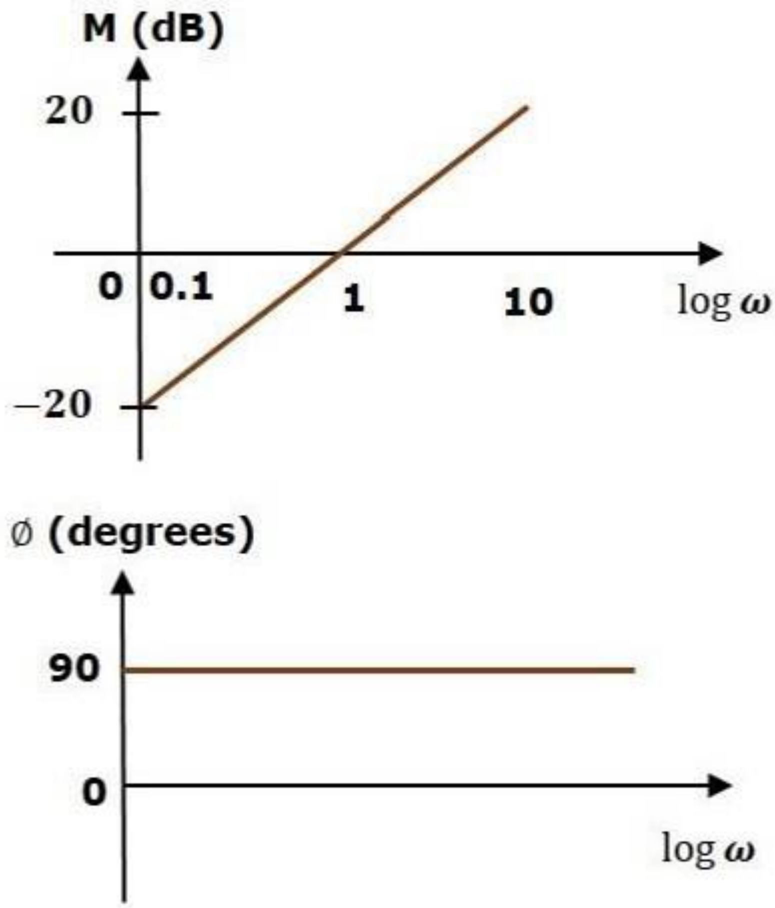
ConsidertheopenlooptransferfunctionG(s)H(s)=sG(s)H(s)=s.
MagnitudeM=20logωM=20log⁡ωdB

Phase angle φ=900φ=900

Atω=0.1ω=0.1rad/sec,themagnitudeis-20 dB.Atω=1ω=1rad/sec,themagnitudeis0dB.

Atω=10ω=10rad/sec,themagnitudeis20dB.

ThefollowingfigureshowsthecorrespondingBodeplot.



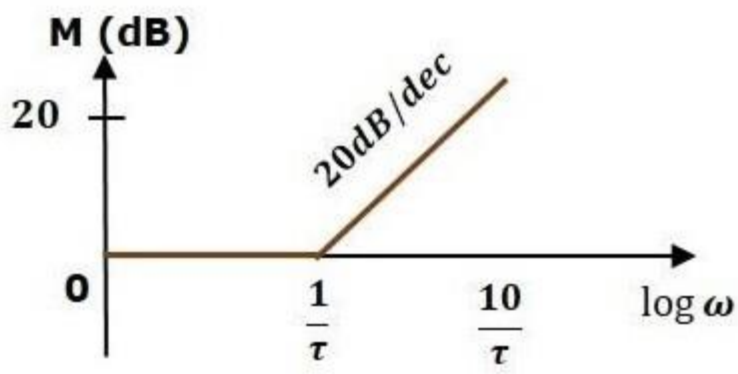
The magnitude plot is a line, which is having a slope of 20 dB/dec. This line startedat ω=0.1ω=0.1 rad/sec having a magnitude of -20 dB and it continues on the same slope. It is touching 0 dB line at ω=1ω=1 rad/sec. In this case, the phase plot is90^0line.

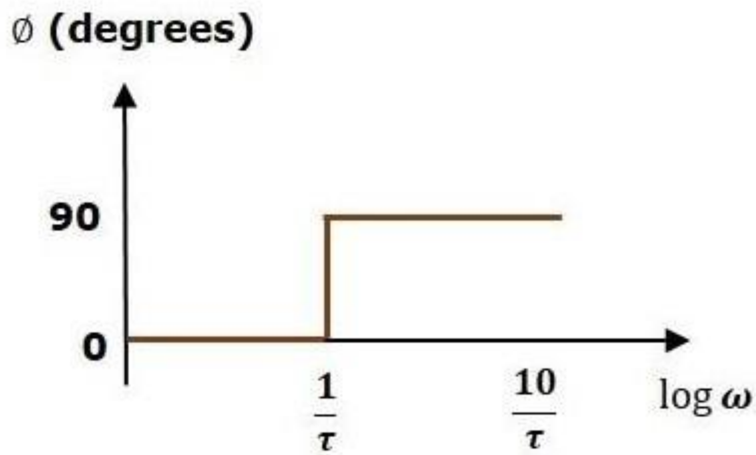
ConsidertheopenlooptransferfunctionG(s)H(s)=1+sτG(s)H(s)=1+sτ.
Magnitude M=20log1+ω2τ2−−−−−−−√M=20log1+ω2τ2 dB

Phaseangleφ=tan−1ωτφ=tan−1⁡ωτdegrees

Forω<1τω<1τ,themagnitudeis0dBandphaseangleis0degrees.

Forω>1τω>1τ,themagnitudeis20logωτ20log⁡ωτdBandphaseangleis 90^0.
The following figure shows the corresponding Bode plot.





The magnitude plot ishaving magnitude of 0 dB upto ω=1τω=1τ rad/sec. From ω=1τω=1τ rad/sec, itishaving a slope of 20 dB/dec. In this case, the phase plot ishaving phase angle of 0 degrees uptoω=1τω=1τrad/secandfromhere,itishavingphaseangleof90^0.ThisBodeplotiscalledthe **asymptotic Bode plot**.

As themagnitude andthe phase plots arerepresentedwith straight lines,theExact Bodeplots resemble the asymptotic Bode plots. The only difference is that the Exact Bode plots will have simple curves instead of straight lines.

Similarly,youcandrawtheBodeplotsforothertermsoftheopenlooptransferfunctionwhich are given in the table.

RulesforConstructionofBodePlots

FollowtheseruleswhileconstructingaBode plot.

- Representtheopenlooptransferfunctioninthestandardtimeconstantform.
- Substitute,s=jωs=jωintheaboveequation.
- Findthecornerfrequenciesandarrangethemin ascendingorder.
- ConsiderthestartingfrequencyoftheBodeplotas1/10thoftheminimumcornerfrequencyor0.1rad/secwhicheveris smaller value and draw the Bode plot upto 10 times maximum corner frequency.

- Drawthemagnitudeplotsforeachtermandcombinetheseplotsproperly.
- Drawthephaseplotsforeachtermandcombinetheseplots properly.

Note−Thecornerfrequencyisthefrequencyatwhichthereisachangeintheslopeofthe magnitude plot.

Example

Considertheopenlooptransferfunctionofaclosedloopcontrolsystem

$$G(s)H(s)=\frac{10}{s(s+2)(s+5)}$$

Letusconvertthisopenlooptransferfunctionintostandardtimeconstantform.

$$G(s)H(s)=\frac{10}{s2\left(\frac{s}{2}+1\right)5\left(\frac{s}{5}+1\right)}$$

$$\Rightarrow G(s)H(s)=\frac{1}{s\left(1+\frac{s}{2}\right)\left(1+\frac{s}{5}\right)}$$

So,wecandrawtheBodeplotinsemilogsheetusingtherulesmentionedearlier

7.4. Minimumphasesystem.

- A transfer function $G(s)$ is minimum phase if both $G(s)$ and $1/G(s)$ are causal and stable.
- Roughly speaking it means that the system does not have zeros or poles on the right-half plane. Moreover, it does not have delay.
- Bode discovered that the phase can be uniquely derived from the slope of the magnitude for minimum-phase system. **Bode's Relation**

Basic Factor	Mag Slope(Low Freq)	Phase(Low Freq)	Mag Slope(High Freq)	Phase (High Freq)
K	0	0	0	0
s^N	20N	90N	20N	90N
$1/(\tau s+1)$	0	0	-20	-90
$1/((s/\omega_n)^2+2\zeta(s/\omega_n)+1)$	0	0	-40	-180

1.6 Non Minimum Phase System: Unstable Zeros

7.5. GainMarginandphasemargin

GainmarginGM isequaltonegativeofthemagnitudeindBatphasecrossoverfrequency.

$$GM=20\log\left(\frac{1}{M_{pc}}\right)=20\log M_{pc}$$

Where,$M_{pc}M_{pc}$isthemagnitudeatphasecrossoverfrequency.Theunitofgainmargin(GM) is **dB**.

PhaseMargin

TheformulaforphasemarginPM$_{PM}$is

$$PM=180^0+\varphi_{gc}PM=1800+\varphi gc$$

Where,$\varphi_{gc}\varphi_{gc}$isthephaseangleatgaincrossoverfrequency.Theunitofphasemarginis **degrees**.

Thestabilityofthecontrolsystembasedontherelationbetweengainmarginandphase margin is listed below.

- IfboththegainmarginGMGMandthephasemargin PMPMarepositive,thenthecontrolsystemis **stable**.
- Ifboththegainmargin GMGMandthephasemargin PMPMareequaltozero,thenthecontrolsystem is **marginally stable**.
- IfthegainmarginGMGMand/orthephasemarginPMPMare/isnegative,thenthecontrolsystemis**unstable**.

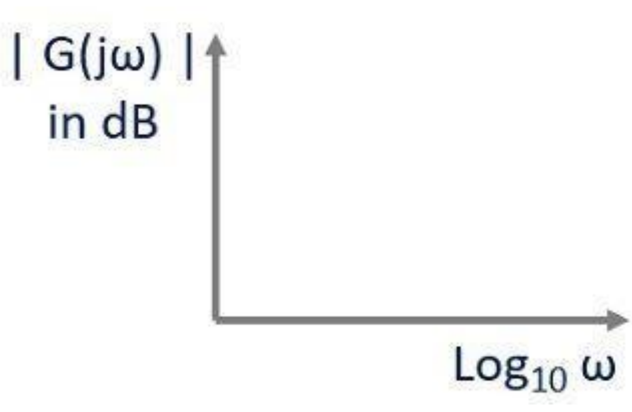
7.6. logmagnitudeversusphase plot

MagnitudePlot:Inthisplot,magnitudeisrepresentedinlogarithmicvaluesagainst logarithmic values of frequency.

ForthetransferfunctionG(jω)H(jω),inordertoexpressthemagnitudeinlogarithmic values, we need to find,

$$| G(j\omega) | = 20 \log_{10} | G(j\omega) | \text{ dB}$$

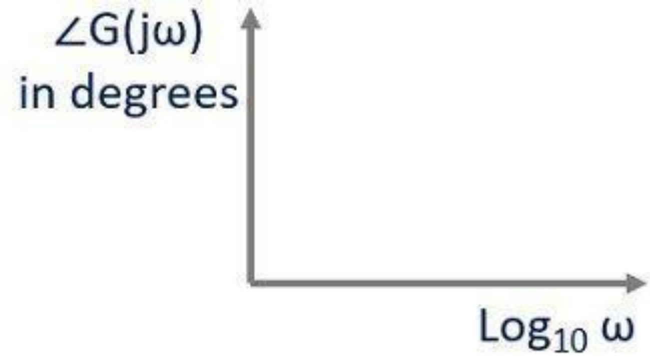
AndthismagnitudeindBisplottedfor$\log_{10}\omega$.Thisisrepresentedinthegeneral



representationfiguregivenbelow:

2.PhaseAnglePlot:Here,thephaseangleindegreesissketchedagainstlogarithmic values of frequency.

Here,theangularvalueofG(jω)indegreesissketchedagainstlog$_{10}$ω.Thefigurehere represents the general representation of phase angle plot:



BodePlotisalsoknownasthelogarithmicplotasitissketchedonthelogarithmicscale and represents a wide range of variation in magnitude and phase angle with respect to frequency, separately. Thus, the bode plots are sketched on semi-log graph paper.

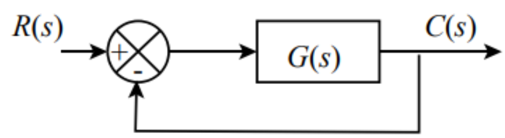
Also,aswecanseethatinboththeplotsthelogarithmicvalueoffrequencyisscaledon the x-axis, so, x-axis can be kept common and both magnitude and phase angle plots can be drawn on the same log paper.

It is to be noted here that, suppose, we are having open-loop transfer function of the systemG(jω)H(jω)andwehavetodeterminetheclosed-loopstabilitybymakinguseof frequencyresponseoftheopen-loopsystem.Then,notsimplyG(jω)butmagnitudeand phaseangleofG(jω)H(jω)istobeplottedagainstlog$_{10}$ω.

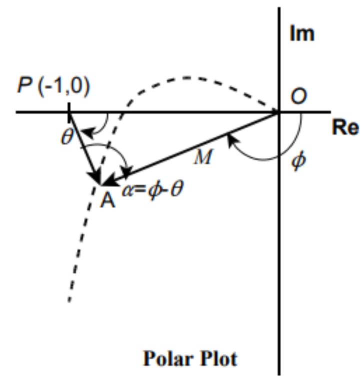
7.7. ClosedLoopFrequencyResponse

TheBodeplotisgenerallyconstructedforanopenlooptransferfunctionofasystem.Inorderto draw theBode plotforaclosedloopsystem,thetransferfunctionhastobedeveloped,andthendecomposedintoitspolesand zeros. This process is tedious and cannot be carried out without the ais of a very powerful calculator or a computer.

With reference to the generic unity feedback system block diagram and its polar plot; the system transfer function is given by:

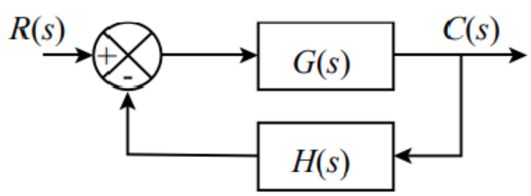


$$\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)}$$

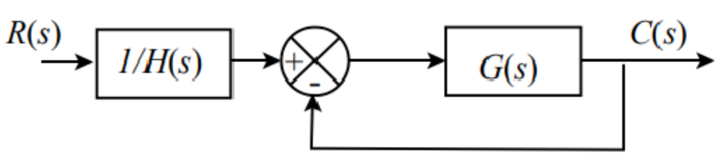


The dashed line in the polar plot is the trace of the tip of the vector *OA* which represents the system. The length of the vector measure the magnitude of the system at a given frequency ω, and the angle ϕ represents the phase shift.

Consider the following non unity feedback system:

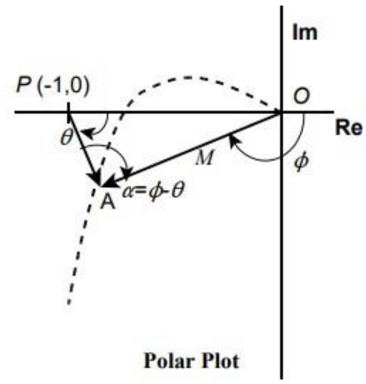


It can be transformed into the following system composed of a simple block cascaded to a unity feedback system:



The frequency response can be then obtained using the additive feature of the Bode plots.

With reference to the polar plot;



$\overrightarrow{OA}$ represents $G(j\omega)$

$\overrightarrow{OP}$ represents -1

But $\overrightarrow{PA} = \overrightarrow{PO} + \overrightarrow{OA}$

$$= 1 + G(j\omega)$$

$$\therefore \frac{\overrightarrow{OA}}{\overrightarrow{PA}} = \frac{G(j\omega)}{1+G(j\omega)}$$

The phase shift angle of the closed loop system is the included angle between the vectors *OA* and *PA*, i.e.;

$$\alpha = \phi - \theta$$

FREQUENCY RESPONSE ANALYSIS
Closed Loop Constant Magnitude Locii

Since the open loop transfer function is a complex quantity that can be expressed as:

$$G(j\omega) = X + jY$$

It follows that the magnitude of the closed loop system can be expressed as follows:

$$M = \frac{|G(j\omega)|}{|1+G(j\omega)|}$$

$$= \frac{|X + jY|}{|(1+X) + jY|}$$

$$\therefore M^2 = \frac{X^2 + Y^2}{(1+X)^2 + Y^2}$$

FREQUENCY RESPONSE ANALYSIS
Closed Loop Constant Magnitude Locii

Expanding and collecting terms, the previous equation gives:

$$X^2\left(1-M^2\right)-2M^2X-M^2+\left(1-M^2\right)Y^2=0$$

For M=1 the above equation reduces to;

For M=1 the above equation reduces to;

$$X = -\tfrac{1}{2}$$

For $M \neq 1$, the equation can be written in the form:

$$X^2 + \frac{2M^2}{M^2-1}X + \frac{M^2}{M^2-1} + Y^2 = 0$$

The preceding equation canot be factored as it stands. However by adding;

$$\frac{M^2}{\left(M^2-1\right)^2}$$

to both sides and factoring gives:

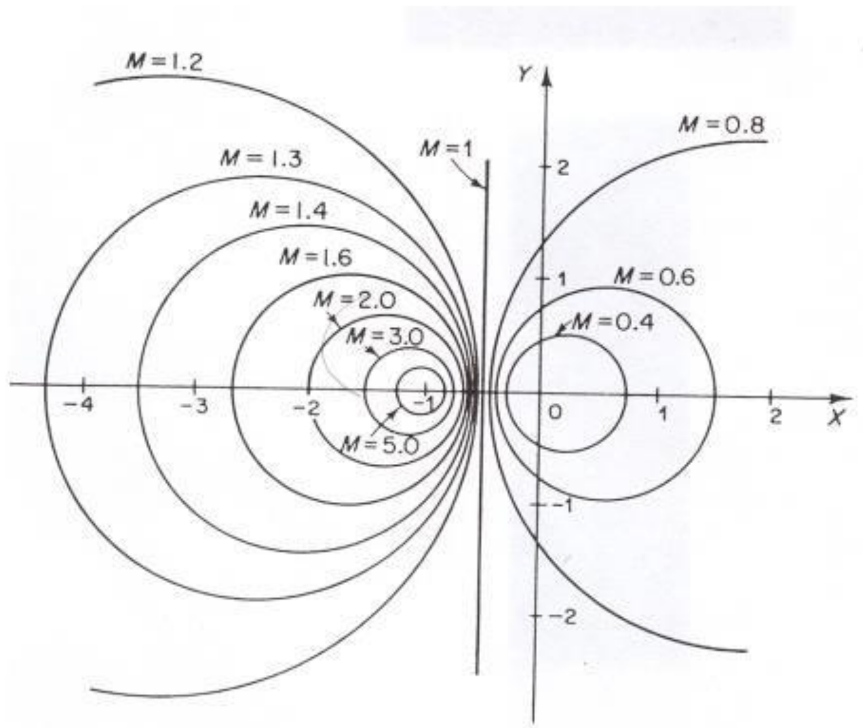
$$\left(X+\frac{M^2}{M^2-1}\right)^2+Y^2=\frac{M^2}{\left(M^2-1\right)^2}$$

This is an equation of a circle with radius and center as follows:

center: $\left(-\frac{M^2}{M^2-1},\ 0\right)$ and radius: $\frac{M}{M^2-1}$

FREQUENCY RESPONSE ANALYSIS
Closed Loop Constant Magnitude Locii

: plot of the previous equation is shown below for different magnitudes M:



Shs

FREQUENCY RESPONSE ANALYSIS
Closed Loop Constant Phase Locii

The phase of the closed loop system can be expressed as follows:

$$\underline{e^{j\alpha}} = \underline{\left| \frac{X+jY}{1+X+jY} \right.}$$

$$\therefore \alpha = \tan^{-1}\frac{Y}{X} - \tan^{-1}\frac{Y}{1+X}$$

Now let $\alpha = N$, then;

$$N = \tan\left[\tan^{-1}\frac{Y}{X} - \tan^{-1}\frac{Y}{1+X}\right]$$

Shortquestionsandans

1Q.whatdoyoumeanbyfrequency response?

Itisdefinedasthesteadystateresponseofasystemduetosinusoidal input

2Q.definebodeplot?

ABodeplotisgenerallyusedinelectricalengineeringandcontroltheoryandisrepresentedbyagraphdepicting the frequency responsesofa particular system. It isan importanttools used in linear time invariantsystems(LTI systems) for showing its gain or the magnitude and the phase response with respect to different operating frequencies.

3Q.Definegainmargin?

hegreaterthe**GainMargin**(GM),thegreaterthestabilityofthesystem.Thegainmarginreferstotheamountof gain, which can be increased or decreased without making the system unstable. It is usually expressed as a magnitude in dB.
We canusually read thegainmargindirectly from theBode plot(as shown inthediagram above). This isdone by calculatingtheverticaldistance betweenthemagnitudecurve(ontheBodemagnitudeplot)andthex-axisatthe frequency where the Bode phase plot = 180°. This point is known as the **phase crossover frequency**.

4QDefinePhasemargin?

Thegreaterthe **PhaseMargin**(PM),thegreaterwillbethestabilityofthesystem.Thephasemarginrefersto the amount of phase, which can be increased or decreased without making the system unstable. It is usually expressed as a phase in degrees.

Longquestions

1. Explaincloseloopfrequencyresponseanalysis?
2. Explainpolarplotanditsruletoconstructpolarplot?
3. Explainallpassminimumphaseequation?

Chapter-8

NyquistPlot

8.1. principleofargument

The Nyquist stability criterion works on the **principle of argument**. It states that if there are P polesandZzerosareenclosedbythe's'planeclosedpath,thenthe corresponding$G(s)H(s)$G(s)H(s)plane must encircle the origin$P-Z$P−Ztimes. So, we can write the number of encirclements N as,

$$N=P-Z$$N=P−Z

- If the enclosed 's' plane closed path contains only poles, then the direction of the encirclement in the$G(s)H(s)$G(s)H(s)plane will be opposite to the direction of the enclosed closed path in the 's' plane.
- If the enclosed 's' plane closed path contains only zeros, then the direction of the encirclement in the $G(s)H(s)$G(s)H(s)plane will be in the same direction as that of the enclosed closed path in the 's' plane.

Let us now apply the principle of argument to the entire right half of the 's' plane by selecting it as a closed path. This selected path is called the **Nyquist**contour.

We knowthat the closed loop controlsystem is stable ifallthe poles of the closed loop transfer function are in the left half of the 's' plane. So, the poles of the closed loop transfer function are nothing but the roots of the characteristic equation. As the order of the characteristic equation increases, it is difficult to find the roots. So, let us correlate these roots of the characteristic equation as follows.

- The Poles of the characteristic equation are same as that of the poles of the open loop transfer function.
- The zeros of the characteristic equation are same as that of the poles of the closed loop transfer function.

Weknowthattheopenloopcontrolsystemisstableifthereisnoopenlooppoleinthetheright half of the 's' plane.

i.e.,$P=0\Rightarrow N=-Z$P=0⇒N=−Z

We know that the closed loop control systemis stable if there is no closed loop pole in theright half of the ‘s’ plane.

i.e.,Z=0⇒N=P

8.2. Nyquiststabilitycriterion

Nyquist stabilitycriterionstatesthenumberofencirclementsaboutthecriticalpoint(1+j0)mustbe equal to the poles of characteristic equation, which is nothing but the poles of the open loop transfer function in the right half of the ‘s’ plane. The shift in origin to (1+j0) gives the characteristic equation plane.

8.3. Nyquiststabilitycriterionappliedtoinversepolarplot

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(TheNyquiststabilitycriterioncanbeappliedequallywelltoinversepolarplots.Themathematicalderivationof the Nyquist stability criterion for inverse polar plots is the same as that for direct polar plots.)

TheinversepolarplotofG(jω)H(jω)isagraphof1/[G(jω)H(jω)]asafunctionofω.Forexample,ifG(jω)H(jω) is

$$G(j\omega)H(j\omega) = \frac{j\omega T}{1 + j\omega T}$$

then

$$\frac{1}{G(j\omega)H(j\omega)} = \frac{1}{j\omega T} + 1$$

Theinversepolarplotforω≥0isthelowerhalfoftheverticallinestartingatthepoint(1,0)ontherealaxis.

The Nyquist stability criterion applied to inverse plots may be stated as follows: For a closed-loop system to be stable,theencirclement,ifany,ofthe-1j0pointbytheII[G(s)H(s)]locus(assmoves alongtheNyquistpath)must be counterclockwise, and the number of such encirclements must be equal to the number of poles of 1/[G(s)H(s)] [that is, the zeros of G(s)H(s)] that lie in the right-half s plane. [The number of zeros of G(s)H(s) in the right-half s plane may be determined by use of the Routh stability criterion.]

Iftheopen-looptransferfunctionG(s)H(s)hasnozerosintheright-halfsplane,thenforaclosed-loopsystem to be stable the number of encirclements of the -1 j0 point by the 1/[G(s)H(s)] locus must be zero.

NotethatalthoughtheNyquiststabilitycriterioncanbeappliedto inversepolarplots, ifexperimentalfrequency-response data are incorporated, counting the number of encirclements of the 1/[G(s)H(s)] locus may be difficult because the phase shift corresponding to the infinite semicircular path in the s plane is difficult to measure. For example, if the open-loop transfer function G(s)H(s) involves transport lag such that

$$G(s)H(s) = \frac{Ke^{-j\omega L}}{s(Ts + 1)}$$

thenthenumberofencirclementsofthe -1j0pointbythe1/[G(s)H(s)]locusbecomesinfinite,andtheNyquist stability criterion cannot be applied to the inverse polar plot of such an open-loop transfer function.

Ingeneral,ifexperimentalfrequency-responsedatacannotbeputintoanalyticalform,boththeG(jω)H(jω)and 1/[G(jω)H(jω)] loci must be plotted. In addition, the number of right-half plane zeros of G(s)H(s) must be determined.It ismore difficult to determine the right-halfplane zerosof G(s)H(s)(inotherwords,to determine whether a given component is minimum phase) than it is to determine the right-half plane poles of G(s)H(s) (in other words, to determine whether the component is stable).

Depending on whether the data are graphical or analytical and whether non minimum-phase components are included,anappropriatestabilitytestmustbeusedformultiple-loopsystems.Ifthedataaregiveninanalytical form or if mathematical expressions for all the components are known, the application of the Nyquist stability criteriononto inversepolarplotscauses no difficulty,andmultiple-loopsystemsmay beanalyzedanddesignedin the inverse GH plane

#

8.4. **EffectofadditionofpolesandzerostoG(S)H(S)ontheshapeofNiquistplot**

- Typically, the Nyquist path should not go through any pole or zero. Hence, the Nyquist path should be slightly modified to avoid this situation.
- Nyquist path is altered by allowing a semi-circle detour with an infinitesimal radius around the origin.
- The small semi-circle is represented using magnitude and phase $\epsilon e^{j\vartheta}$.
  - Note that for type-1 systems, $\lim_{s \to \epsilon e^{j\vartheta}} GH(s) = \frac{1}{\epsilon} e^{-j\vartheta}$
  - Note that for type-1 systems, $\lim_{s \to \epsilon e^{j\vartheta}} GH(s) = \frac{1}{\epsilon^2} e^{-j2\vartheta}$
- Example $G(s) = K/[s(1+Ts)]$

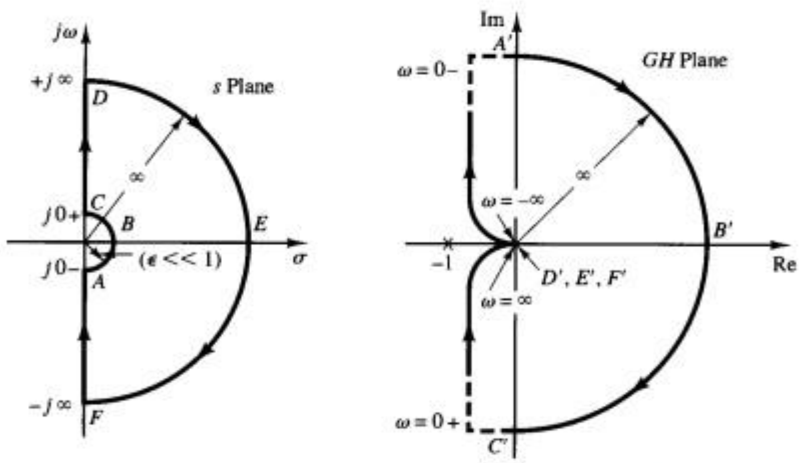


Figure 11: Modified Nyquist Path GH(s)=$K/[s(1+Ts)$

- P=0,
- No encirclements form contour mapping N=0,
- Z=P+N=0 ⇒the system is stable.

Example: $G(s) = K/[s^2(1+Ts)]$

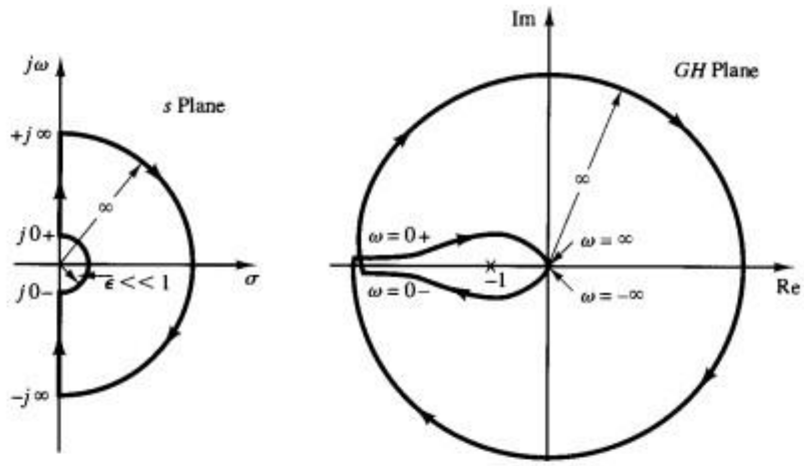


Figure 12: Modified Nyquist Path $G(s) = K/[s^2(1 + Ts)]$

- P=0 for positive T
- Two clockwise encirclements N=2,
- $\therefore$Z=N+P=2 $\Rightarrow$there exist two zeros for the characteristic equations in the RHP.
- Hence, the system is unstable.

8.5. Assesmentofrelativestability

...have introduced the Nyquist criterion for the absolute stability analysis of the system. Using the Nyquist criterion, it is also possible to find the relative stability of the system. By relative stability we mean how close the system is to instability, and we can improve the stability of the system. The degree or extent of the system is called relative stability. If the Nyquist polar plot is close to $-1 + j0$ point, the system is on the verge of the instability. The proximity to $-1 + j0$ point is specified in terms of the following two quantities: (i) Gain margin and (ii) Phase margin

15.6.1 Gain Margin

The gain margin is defined as the reciprocal of the open-loop transfer function evaluated at the frequency (ω_{pc}) at which the phase angle is $-180°$.

$$\text{Gain margin} = \frac{1}{|G(j\omega)H(j\omega)|}$$

and

$$\phi = \angle G(j\omega)H(j\omega).$$

The frequency ω_{pc} is known as the phase crossover frequency at which the polar plot crosses the negative real axis. Gain margin measures the relative distance between the $-1 + j0$ point and the $G(j\omega)H(j\omega)$ plot.

Depending upon the phase crossover point X shown in Fig. 15.8, we set the gain. If the point X is too near the $-1 + j0$ point, we can decide how much to reduce the gain and if the point X is too far from the $-1 + j0$ point, we can decide how much to increase the gain.

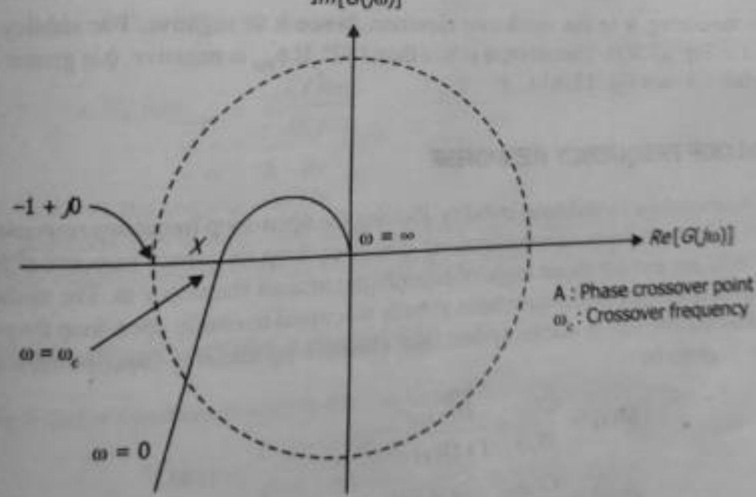
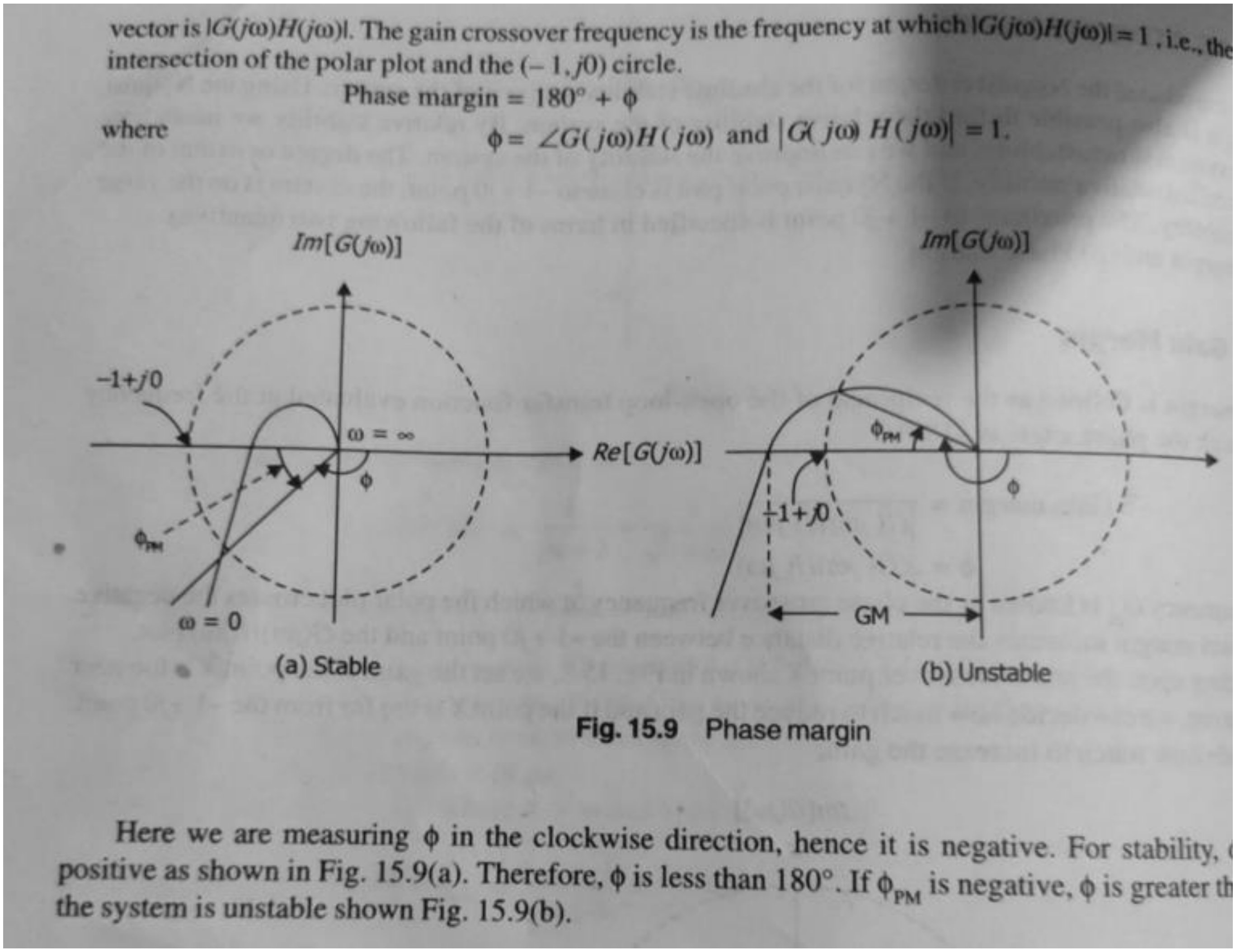


Fig. 15.8 Gain margin

15.6.2 Phase Margin

Gain margin alone is not sufficient to indicate the relative stability of a system. Another parameter, phase margin, supplements the gain margin. A rigorous definition of the phase margin (ϕ_{PM}) is the angle by which the polar plot is rotated to cause it to pass through $(-1, j0)$ point. ϕ_{PM} is the angle between the negative real axis and the radius vector joining the origin to the gain crossover frequency (ω_{gc}). The radius



vector is $|G(j\omega)H(j\omega)|$. The gain crossover frequency is the frequency at which $|G(j\omega)H(j\omega)| = 1$, i.e., the intersection of the polar plot and the $(-1, j0)$ circle.

$$\text{Phase margin} = 180^\circ + \phi$$

where $\phi = \angle G(j\omega)H(j\omega)$ and $|G(j\omega)\ H(j\omega)| = 1$.

Fig. 15.9 Phase margin

Here we are measuring ϕ in the clockwise direction, hence it is negative. For stability, (positive as shown in Fig. 15.9(a). Therefore, ϕ is less than 180°. If ϕ_{PM} is negative, ϕ is greater th the system is unstable shown Fig. 15.9(b).

8.6. ConstantM&Ncircle

ConstantmagnitudelocithatareM-circlesandconstantphaseanglelocithatareN-circlesarethefundamental components

• TheconstantMandconstantNcirclesinG(jω)planecanbeusedfortheanalysisanddesignofcontrol systems.
• HowevertheconstantMandconstantNcircles ingainphaseplanearepreparedforsystemdesignandanalysis as these plots supply information with fewer manipulations.

Gainphaseplaneisthegraphhavinggainindecibelalongtheordinate(verticalaxis)andphaseanglealongthe abscissa (horizontal axis).

• TheMandNcirclesofG(jω) inthegainphaseplanearetransformedinto MandNcontoursinrectangularco- ordinates.

• ApointontheconstantMlociinG(jω) planeistransferredto gainphaseplanebydrawingthevectordirected fromtheoriginof G(jω) plane to a particular pointonM circle andthenmeasuring the length in db andangle in degree.

ThecriticalpointinG(jω),planecorrespondstothepointofzero decibeland -180o inthegainphaseplane.Plot of M and N circles in gain phase plane is known as Nichols chart /plot.

TheNicholsplotisnamedaftertheAmericanengineerN.BNicholswhoformulatedthisplot. Compensators

can be designed using Nichols plot.

Nicholsplottechniqueishoweveralsousedindesigningofdcmotor.Thisisusedinsignalprocessingandcontrol design.

Nyquistplotincomplexplaneshowshowphaseoftransferfunctionandfrequencyvariationofmagnitudeare related.

• Angleofpositiverealaxisdeterminesthephaseanddistancefromoriginofcomplexplanedeterminesthegain.

Advantages

• Gainandphasemargincanbedeterminedeasilyandalso graphically.

• Closedloopfrequencyresponseisobtainedfromopenloopfrequencyresponse.

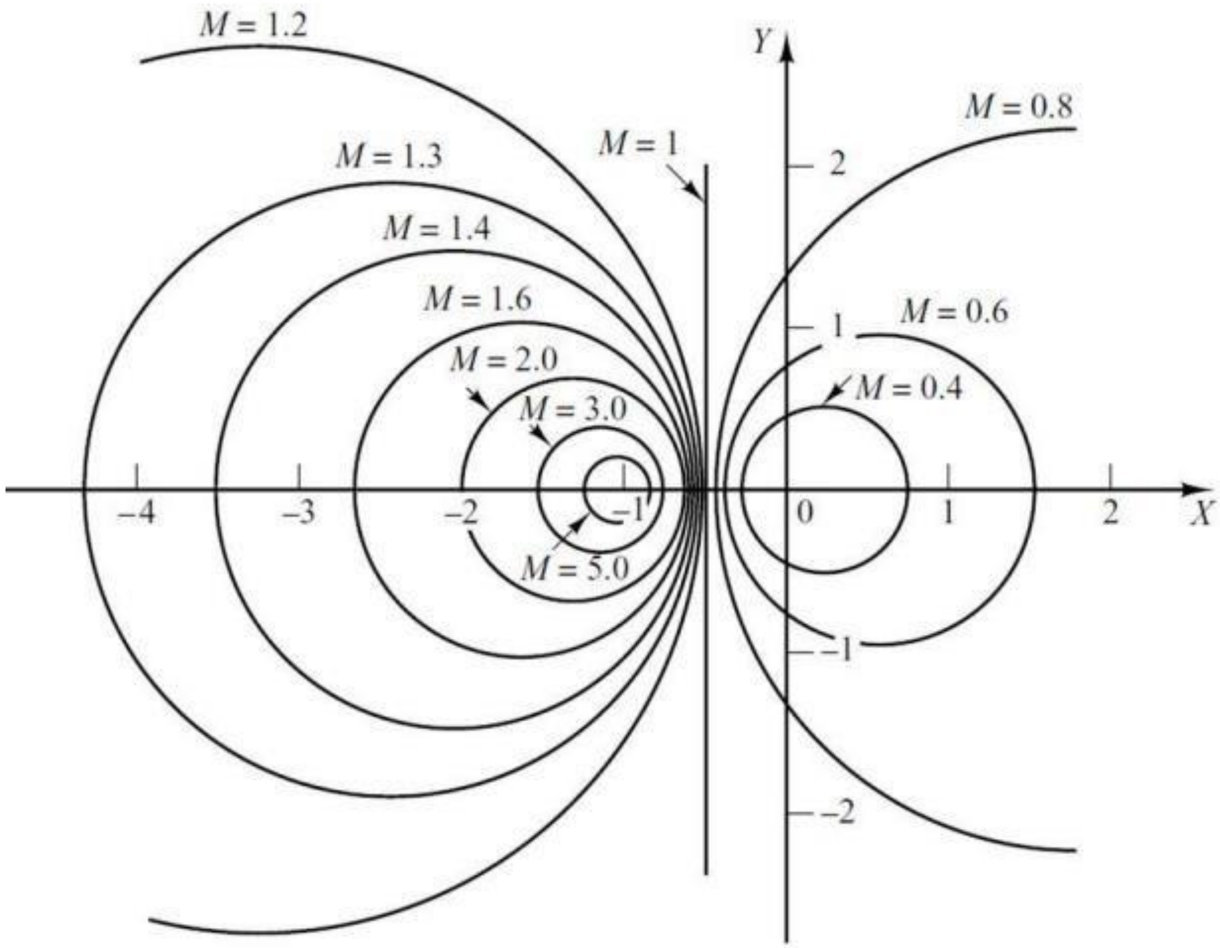
• Gainofthesystemcanbeadjustedtosuitablevalues.

• Nicholschartprovidesfrequencydomainspecifications.

Disadvantage

• UsingNicholsplotsmallchangesingaincannotbeencounteredeasily.

Constant M & N Circles



8.7. NicholasChart

15.8 NICHOL'S CHART

It is possible to obtain the frequency response by sketching the magnitude in dB against the phase angle for various frequencies. The plot obtained is called gain-phase plot or logmagnitude versus phase plot. Fig. 15.13 shows such a gain-phase plot for $G(j\omega) = 1/(1 + j\omega T)$, i.e., $G(s) = 1/(1 + sT)$.

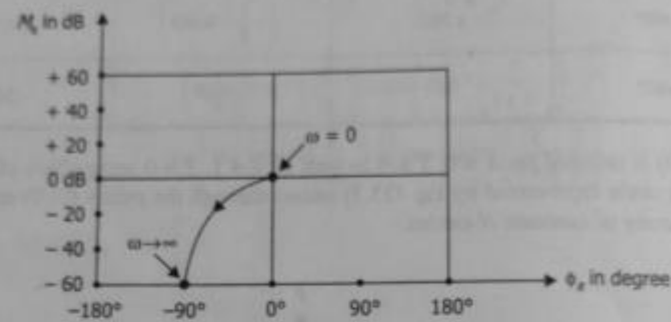


Fig. 15.13 Gain-phase plot for $G(j\omega)$

After transferring the constant magnitude loci (*M* circles) and constant phase angle loci (*N* circles) to the gain phase plot, the resultant chart is known as Nichol's chart.

If we superimpose the gain–phase plot of an open-loop transfer function on Nichol's chart, we get very easily the closed-loop frequency response. The magnitude is expressed in dB while the phase angle is in degrees. Nichol's chart gives the points of intersection of the gain-phase plot of an open-loop transfer function, which are symmetrical about the −180° axis. The *M* and *N* loci also repeat for every 360°. Again symmetry exists at every 180° interval. The *M* loci are centered about the critical point (0 dB, − 180°). Figure 15.14 shows Nichol's chart.

Nichol's chart is used for the following:

(1) The complete closed-loop frequency response can be determined.
(2) The value of resonant peak M_r or closed-loop system, with given $G(j\omega)$ can be obtained.
(3) The frequency ω_r corresponding to the M_r for the closed-loop system can be also obtained.
(4) If M_r and ω_r are known, other frequency and time domain specifications can also be calculated.
(5) The 3 dB bandwidth of the closed-loop system can be determined.
(6) For the given M_r, it is possible to design the value of *K*.
(7) For designing the compensating networks which are useful to meet more than one specifications for the closed-loop system.

Shot questions

1. Defineencircled?

Ans-ifpointisfoundtobeinsidethepath.Thepointissaidtobeencircledbytheclose path

2. Defineanalyticfunction?

Ans-Afunctionissaidto beanalyticatapointinaplane.ifitsvalueandderivativehave finiteexistanceatthat point.

3. whatdoyoumeanbyNyquist criterion?

Ans-Itfocusonrelativestabilityofthesystem.Itispossibleto determinethestabilityofacloselooppolefrom an open loop pole without knowing the roots of close loop system.ANiquist plot is based on a polar plot.

Long Questions

1. ForG(s)H(s)=1/s(s+2)drawtheNiquistplotanddecidestability?

2. ForG(s)H(s)=$1/s^2(s+2)$sketchtheNyquistplotanddeterminethestabilityofthesystem?